

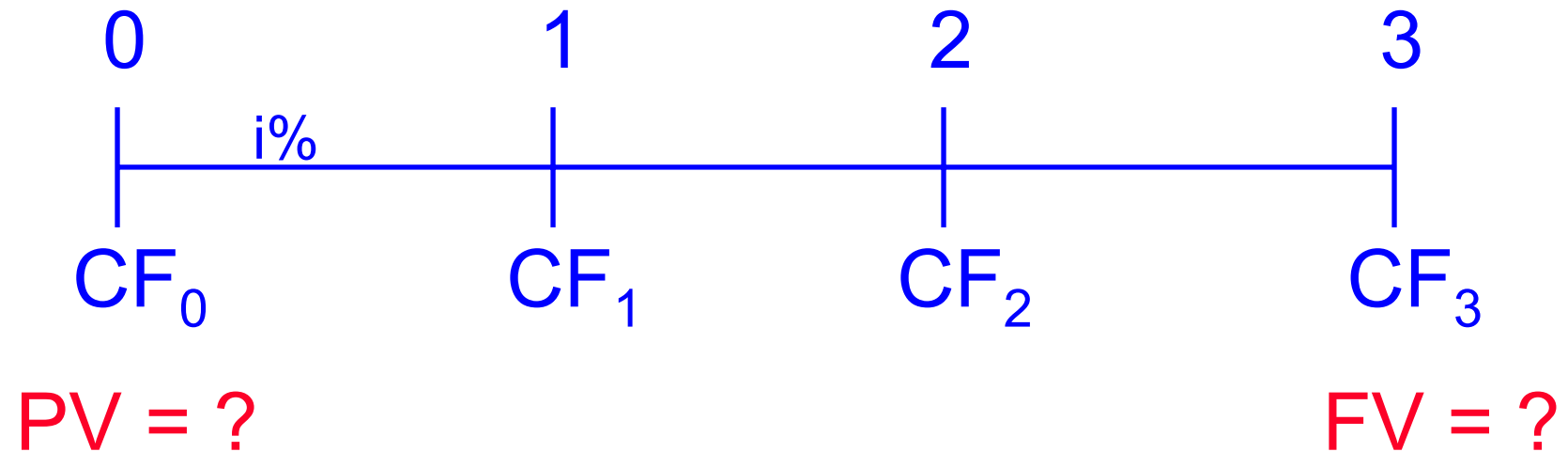
Time Value of Money

- FV and PV of a single cash flow
- FV and PV of cash flows
- Rates of return
- Amortization

FV and PV of A Single Cash Flow

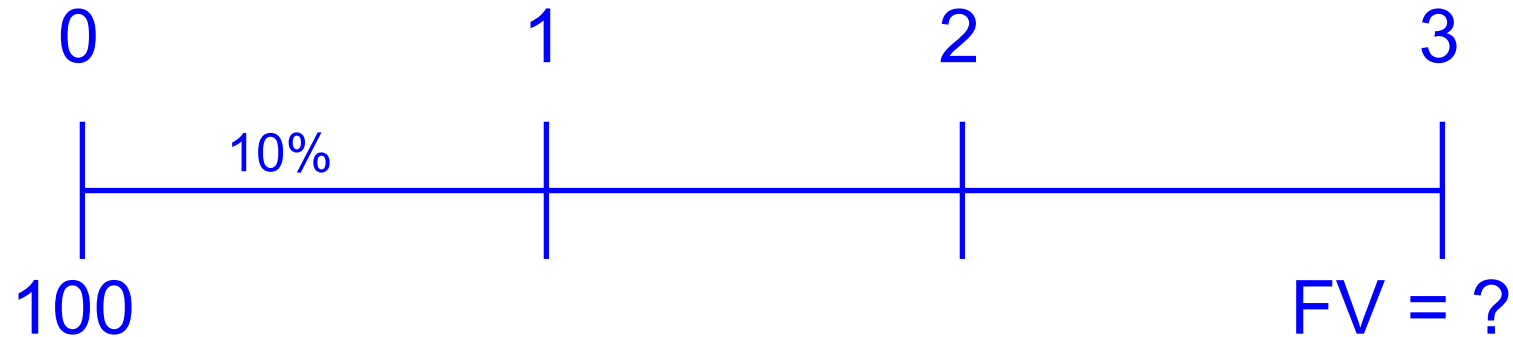
- Solve FV of a single cash flow
- Solve PV of a single cash flow
- Solve number of periods
- Solve interest rate

Time lines show timing of cash flows.

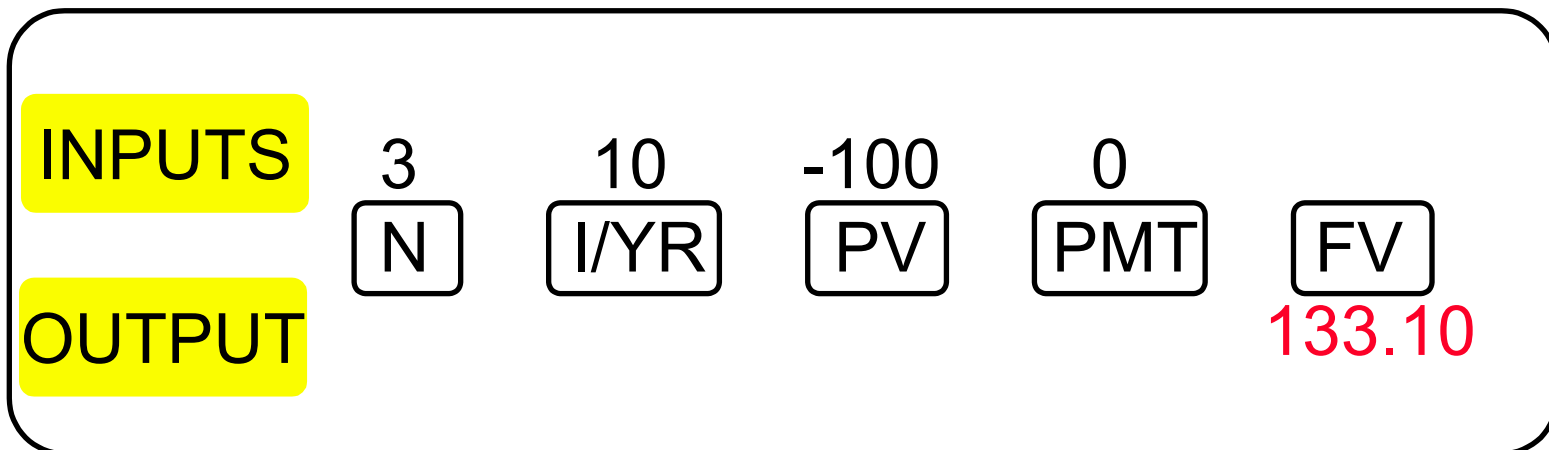


Time 0 is today; Time 1 is the end of Period 1; or the beginning of Period 2.

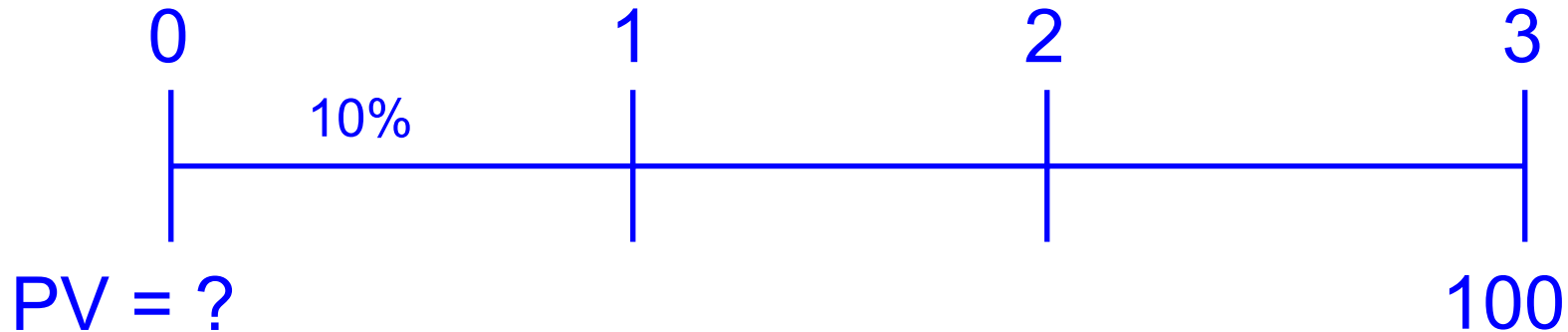
What's the **FV** of an initial \$100 after 3 years if $i = 10\%$?



$FV_n = PV(1 + i)^n$. Finding FVs is compounding.



What's the **PV** of \$100 due in 3 years if $i = 10\%$?



$PV = FV_n / (1 + i)^n$ Finding PVs is discounting.

INPUTS	3	10	0	100
	N	I/YR	PMT	FV
OUTPUT			-75.13	

If sales grow at 20%, **how long** before sales double?

$$FV_n = PV(1 + i)^n$$
$$\$2 = \$1(1.20)^n$$

INPUTS		20	-1	0	2
	N	I/YR	PV	PMT	FV
OUTPUT	3.8				

What interest rate would cause \$100 to grow to \$125.97 in 3 years?

$$\$100 (1 + i)^3 = \$125.97.$$

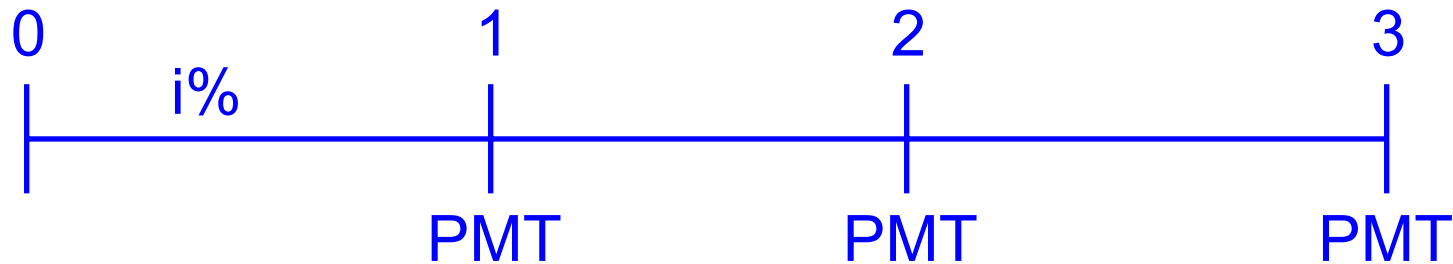
INPUTS	3		-100	0	125.97
	N	I/YR	PV	PMT	FV
OUTPUT		8%			

FV and PV of Cash Flows

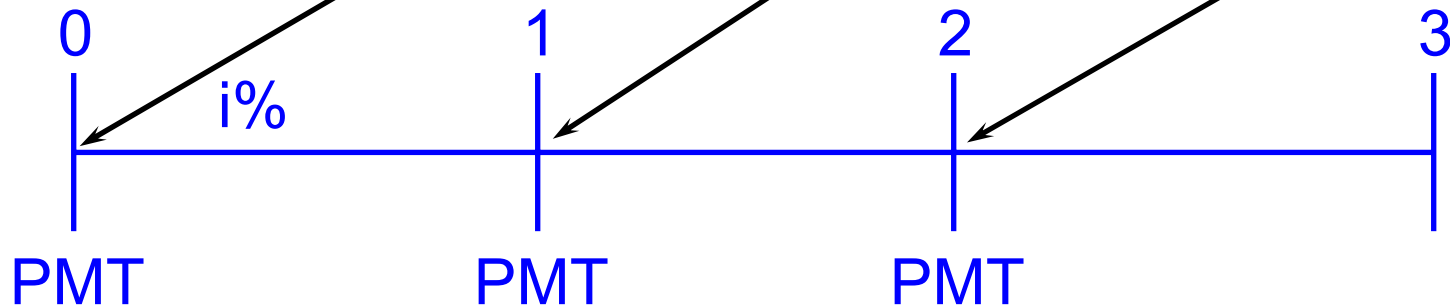
- Ordinary Annuity and Annuity Due
- Uneven cash flows
- Perpetuity

Ordinary Annuity and Annuity Due

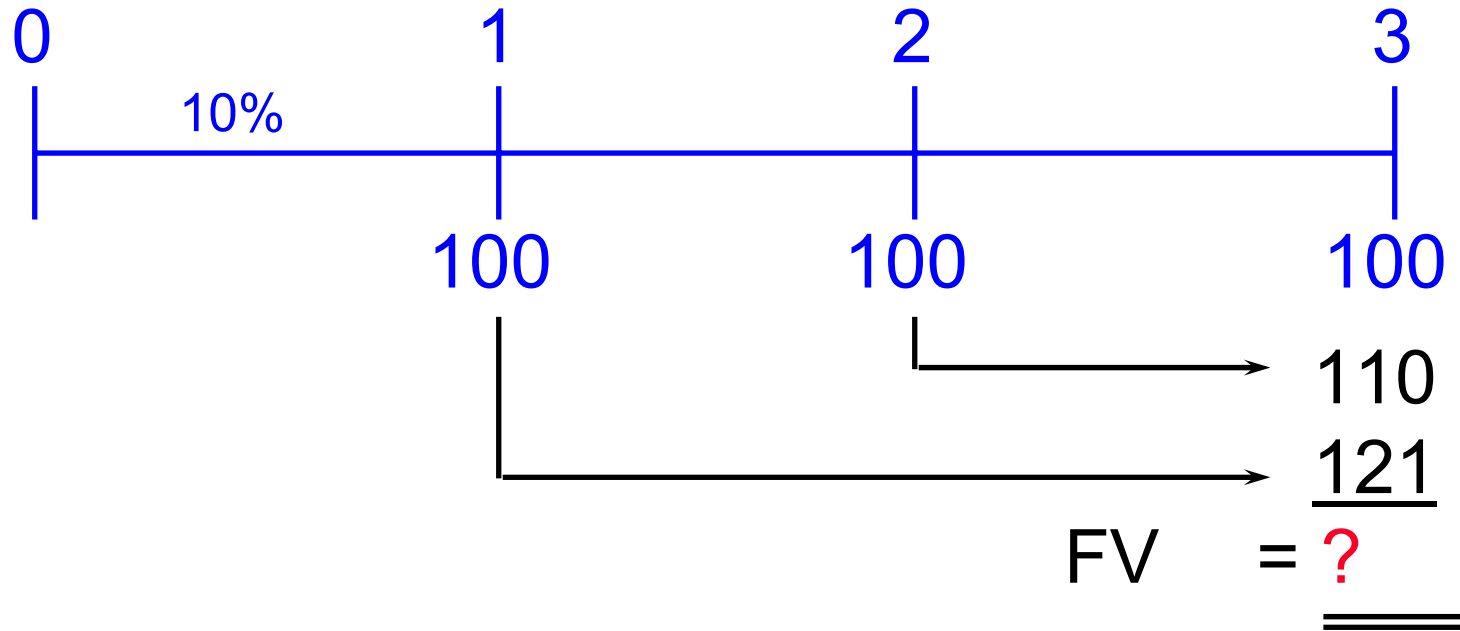
Ordinary Annuity



Annuity Due



What's the FV of a 3-year annuity of \$100 at 10%?



INPUTS

3

10

0

-100

N

I/YR

PV

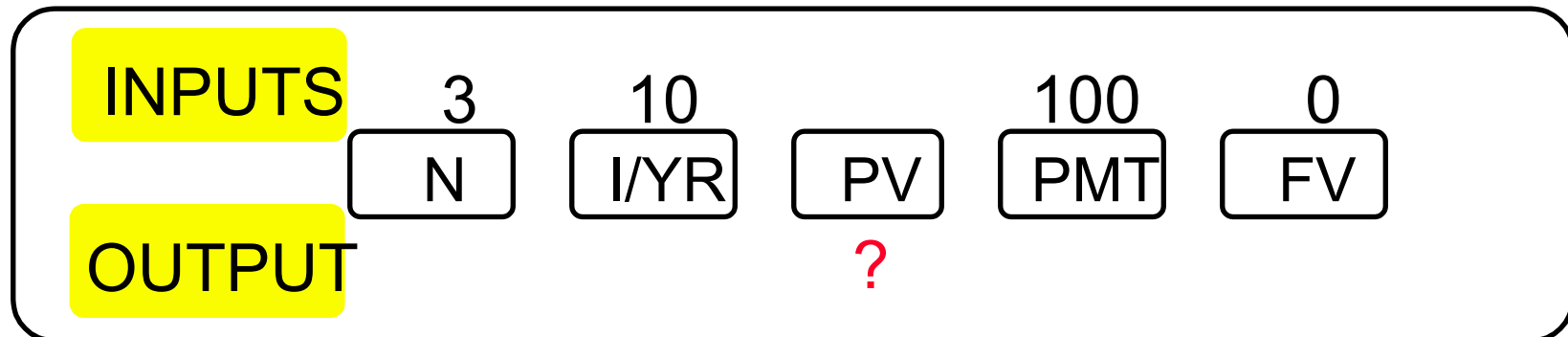
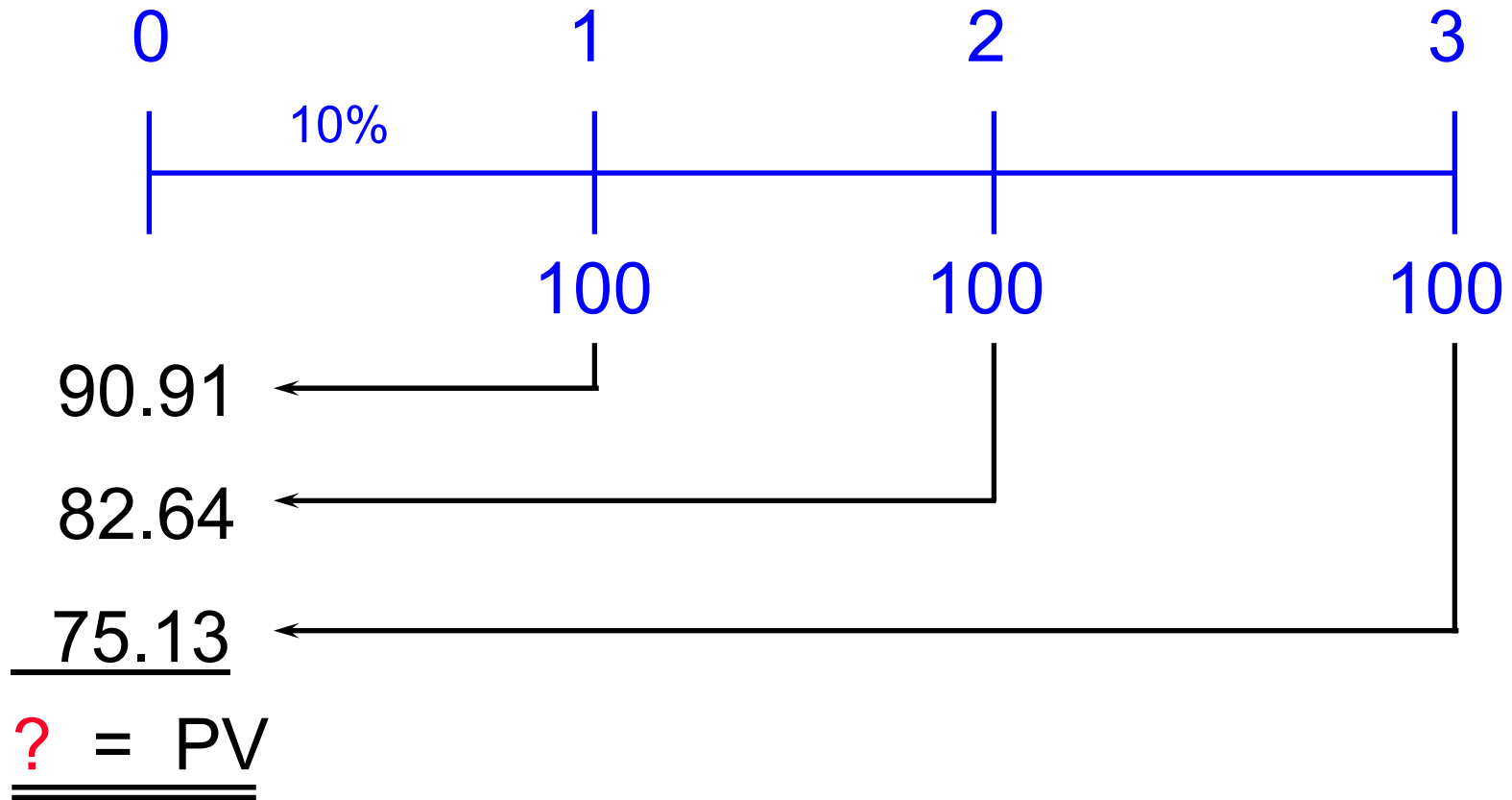
PMT

FV

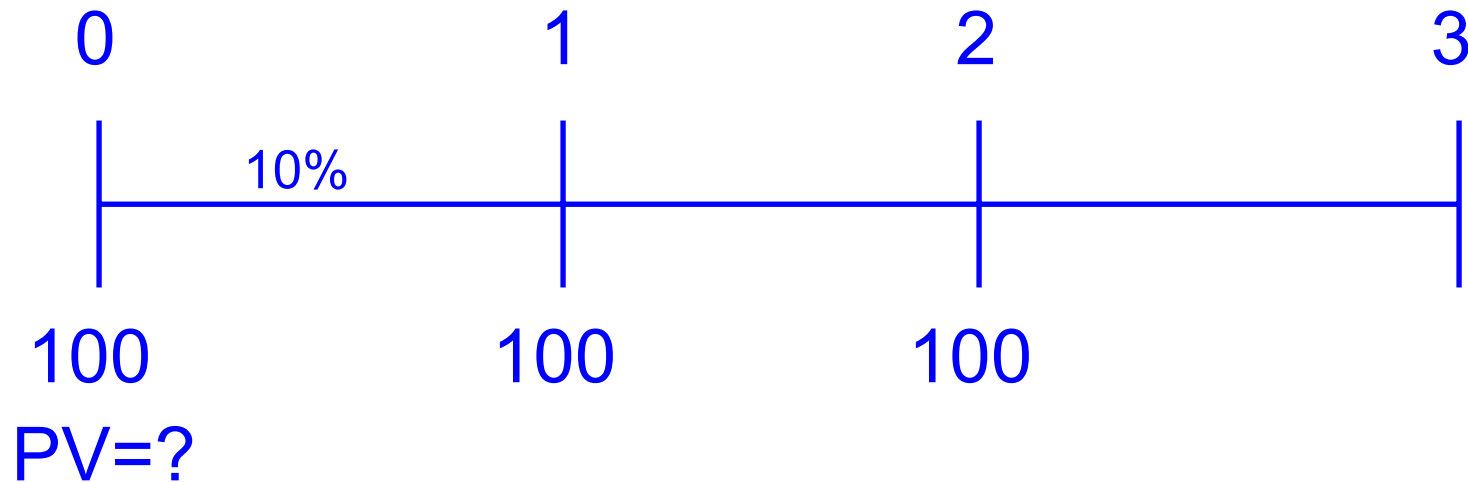
OUTPUT

?

What's the PV of this annuity?



Find the PV if the annuity were an annuity due.



Switch from “End” to “Begin.”

INPUTS

3

10

100

0

N

I/YR

PV

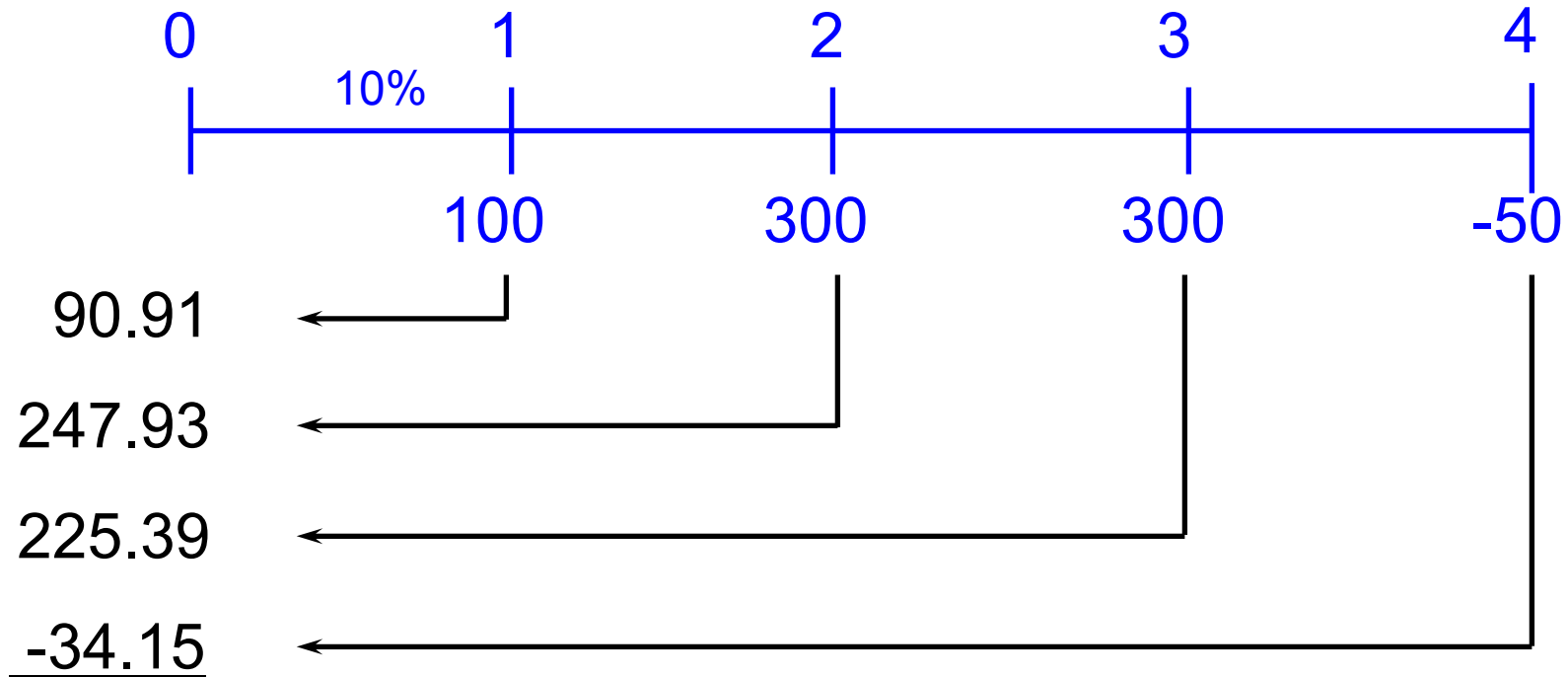
PMT

FV

OUTPUT

?

What is the PV of this uneven cash flow stream?



? = NPV

$$CF_0 = 0 \quad CF_1 = 100 \quad CF_2 = 300 \quad CF_3 = 300 \quad CF_4 = -50$$

Interest = 10

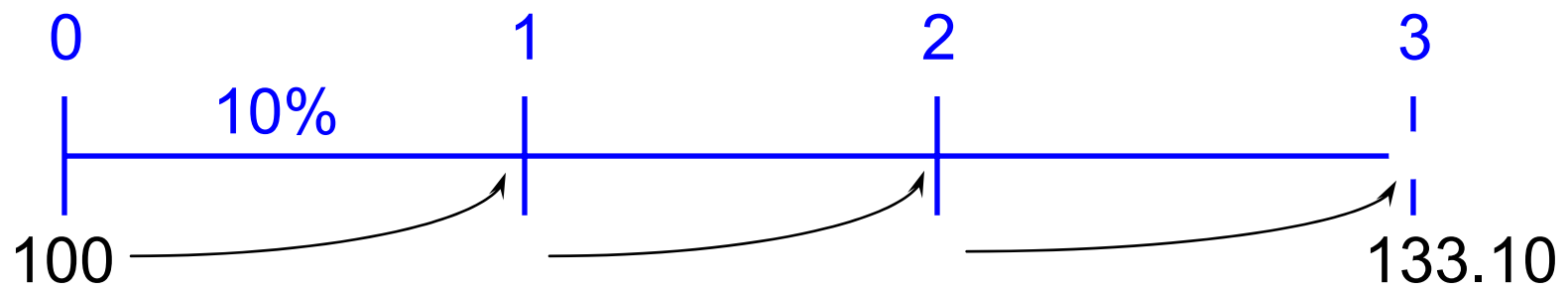
NPV = ?

Rates of Return

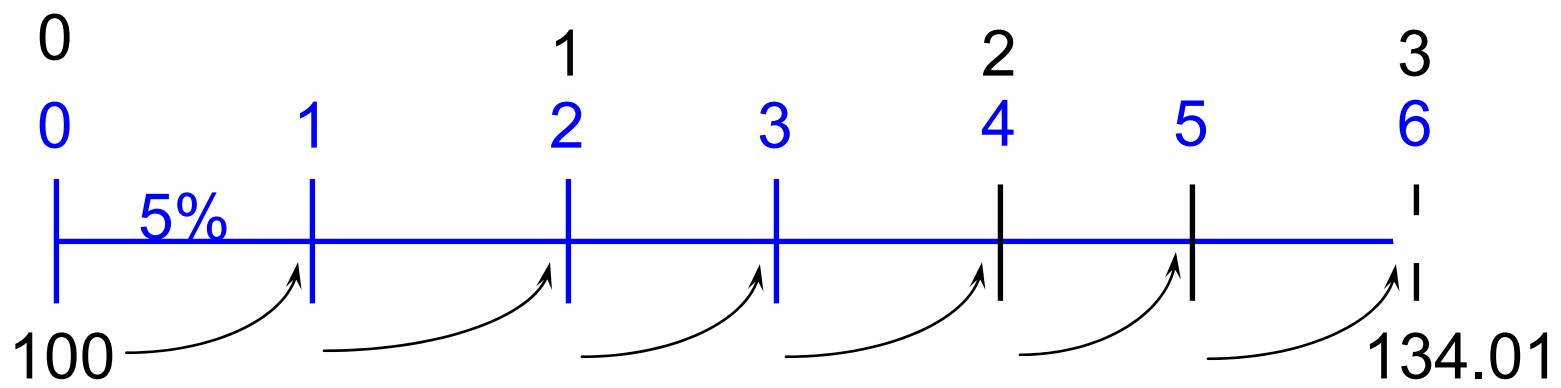
- Compounding frequency
- Three types of rates

Compounding Frequency

Will the FV of a lump sum be larger or smaller if we compound more often, holding interest rate constant?



Annually: $FV_3 = \$100(1.10)^3 = \133.10 .



Semiannually: $FV_6 = \$100(1.05)^6 = \134.01 .

Three Different Rates

i_{Nom} nominal, or stated, or quoted, rate per year

i_{Per} periodic rate

EAR effective annual rate

m : number of compounding periods per year

$$i_{\text{Per}} = i_{\text{Nom}}/m$$

$$\text{EAR} = (1 + i_{\text{Nom}}/m)^m - 1$$

Continuous compounding

$\text{EAR} = e^i - 1$, where e has a value of 2.7183.

10%, compounding annually, quarterly, monthly, daily (365), and continuously?

$$EAR_{\text{Annual}} = 10\%.$$

$$EAR_Q = (1 + 0.10/4)^4 - 1 = 10.38\%.$$

$$EAR_M = (1 + 0.10/12)^{12} - 1 = 10.47\%.$$

$$EAR_{D(365)} = (1 + 0.10/365)^{365} - 1 = 10.516\%.$$

$$EAR_{\text{cont}} = e^{0.1} - 1 = 10.517\%.$$

Solve the FV of \$100 after 3 years under 10% semiannual compounding? Continuously compounding?

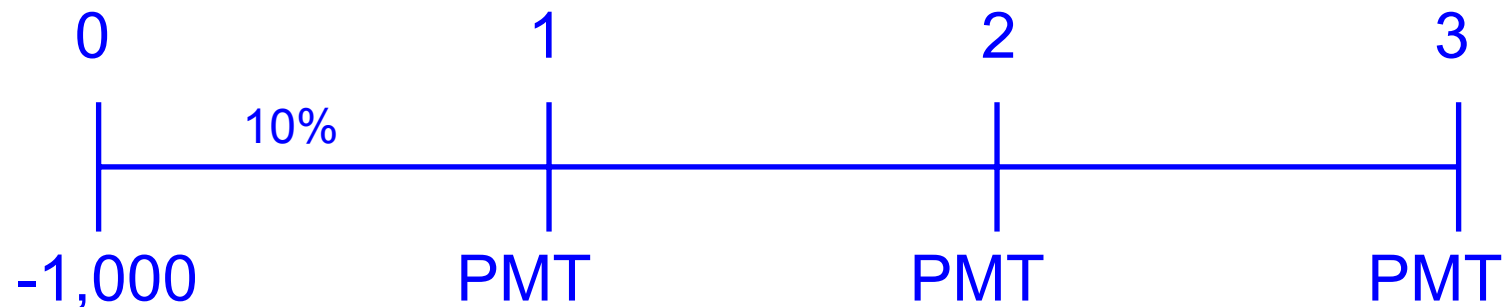
$$FV = \$100 \left(1 + \frac{i_{\text{nom}}}{m} \right)^{m * \text{number of years}}$$

$$FV = \$100 e^{i * \text{number of years}}$$

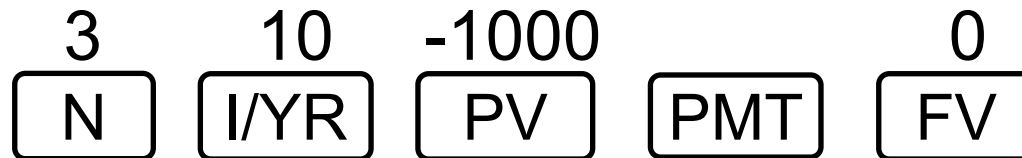
Amortization

Construct an amortization schedule for a \$1,000 loan, 10% annual rate loan with 3 equal payments.

Find the annual payment (or installment)



INPUTS



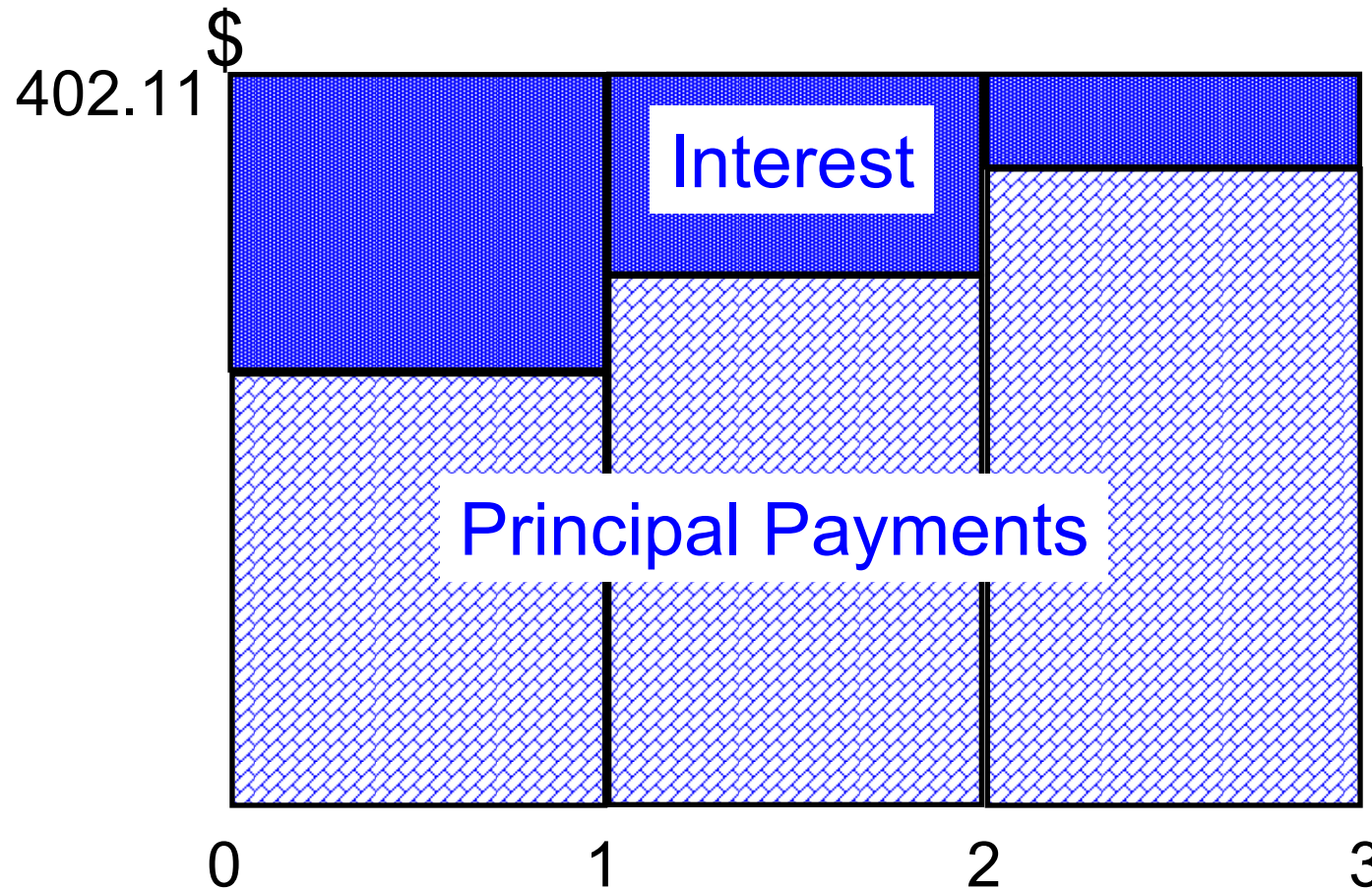
OUTPUT

?

Amortization schedule

int rate 10%

period	beg bal	pmt	int	prin pmt	end bal
		solve annuity pmt	beg bal*int rate	pmt - int	beg bal - prin pmt
0					1000.00
1	1000.00	402.11			
2		402.11			
3		402.11			



Interest declines because outstanding balance declines. Lender earns 10% on loan outstanding, which is falling.