

1. $a_n = a_{n-1} + 3a_{n-2}$

Initial conditions

$$a_0 = 1$$

$$a_1 = 2$$

What are a_2 and a_3 ?

Is $\{a_n\}$ a solution of $a_n = 8a_{n-1} - 16a_{n-2}$ when

2. $a_n = 0$

3. $a_n = 2^n$

4. Find closed form solution for:

$$a_n = a_{n-1} + 4$$

$$a_0 = 2$$

Population growing at 1.3% a year.

5. Recurrence for population after n years.

$$P_n =$$

6. Explicit formula

Initial conditions for 2009.

$$P_0 = 6,800,000,000$$

$$P_1 = 1.013P_0$$

7. Population in 20 years?

$$a_n = 7a_{n-1} - 10a_{n-2} \quad \text{Initial conditions } a_0 = 2, a_1 = 1$$

8. What is c_1, c_2 ?

9. Find the roots, r_1, r_2 $r^2 - c_1r - c_2 = 0$

10. Solve $a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ for α_1 and α_2

11. Solve α_1 and α_2

12. Solve $a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ by substituting

1. $a_n = a_{n-1} + 3a_{n-2}$

$a_0 = 1$

$a_1 = 2$

What are a_2 and a_3 ?

$a_2 = a_{2-1} + 3a_{2-2} = a_1 + 3a_0 = 2 + 3(1) = 5$

$a_3 = a_{3-1} + 3a_{3-2} = a_2 + 3a_1 = 5 + 3(2) = 11$

Is $\{a_n\}$ a solution of $a_n = 8a_{n-1} - 16a_{n-2}$ when

2. $a_n = 0$

a_n	$= 8a_{n-1} - 16a_{n-2}$	
	$= 8(0) - 16(0)$	
	$= 0$	Yes

3. $a_n = 2^n$

a_n	$= 8a_{n-1} - 16a_{n-2}$	
	$= 8(2^{n-1}) - 16(2^{n-2})$	
	$= 2^3(2^{n-1}) - 2^4(2^{n-2})$	
	$= 2^{n+2} - 2^{n+2}$	
	$= 0$	No

4. Find closed form solution for:

$$a_n = a_{n-1} + 4$$

$$a_0 = 2$$

a_n	$= a_{n-1} + 4$
	$= (a_{n-2} + 4) + 4$
	$= (a_{n-3} + 4) + 2*4$
	$= a_{n-3} + 3*4$
	$= a_0 + n*4$
	$= 2 + n*4$

Population growing at 1.3% a year.

5. Recurrence for population after n years.

a. $P_n = P_{n-1} + 0.013P_{n-1} = 1.013P_{n-1}$

6. Explicit formula

Initial conditions for 2009.

$$P_0 = 6,800,000,000$$

$$P_1 = 1.013P_{1-1} = 1.013P_0$$

$$P_2 = 1.013P_1 = (1.013)(1.013)P_0 = 1.013^2P_0$$

$$P_3 = (1.013)P_2 = (1.013) 1.013^2P_0 = 1.013^3P_0$$

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$$P_n = (1.013)P_{n-1} = (1.013)1.013^{n-1}P_0 = 1.013^nP_0$$

7. Population in 20 years?

$$P_{20} = 1.013^{20} 6,800,000,000$$

$$a_n = 7a_{n-1} - 10a_{n-2} \quad \text{Initial conditions } a_0 = 2, a_1 = 1$$

8. What is c_1, c_2 ?

$$c_1 = 7, c_2 = -10$$

9. Find the roots, r_1, r_2 $r^2 - c_1r - c_2 = 0$

$$r^2 - 7r + 10 = 0$$

$$(r-5)(r-2)$$

$$r_1 = 5, r_2 = 2$$

10. Solve $a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ for α_1 and α_2

$$\begin{array}{lcl} a_0 & = & 2 \\ & = & \alpha_1(5)^0 + \alpha_2(2)^0 \\ & = & \alpha_1 + \alpha_2 \\ a_1 & = & 1 \\ & = & \alpha_1(5)^1 + \alpha_2(2)^1 \\ & = & 5\alpha_1 + 2\alpha_2 \end{array}$$

11. Solve α_1 and α_2

$$\begin{array}{lcl} 2\alpha_1 + 2\alpha_2 & = & 4 \\ \alpha_1 + \alpha_2 & = & 2 \\ \hline -(5\alpha_1 + 2\alpha_2) & = & 1 \\ -3\alpha_1 & = & 3 \\ \alpha_1 & = & -1 \end{array}$$

12. Solve $a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ by substituting

$$\begin{array}{l} a_n = \alpha_1 r_1^n + \alpha_2 r_2^n \\ = (-1)(5)^n + (3)(2)^n \end{array}$$