T102 SECTION 6.4 REAL NUMBERS

The Real Numbers consist of the union of the set of Rational Numbers with the set of Irrational Numbers. We know that Rational Numbers are numbers that can be expressed as a repeating decimal or as a terminating decimal. Irrational Numbers are numbers that neither terminate nor repeat.

In other words, Irrational Numbers are numbers that are not rational.

Examples: .374121658239... .02.002000200002000002...

π - the ratio of the circumference of a circle to its diameter, was proven in the mideighteenth century to be irrational. The value of π has been computed to over a trillion decimal places with no apparent pattern.

We use two approximations for π.

One is the fraction, $\frac{22}{7}$. The other is a decimal, 3.14.

REMEMBER-These are **ONLY** *approximations* of π.

PRINCIPAL SQUARE ROOT: If a is any non-negative number, the principal square root of a (denoted $\sqrt{a}$) is the nonnegative number b such that $b^2 = a$.

$$\sqrt{a} = b \text{ if and only if } b^2 = a.$$

$\sqrt{a}$ is a radical $\qquad$ $\sqrt{}$ is a radical sign $\qquad$ a is the radicand

Examples:

$\sqrt{9}$ = _______________ $\sqrt{49}$ = _______________ $\sqrt{\frac{16}{25}}$ = __________

Find the following: a) The square roots of 36 ______________________

b) The principal square root of 36 ______________________

c) $\sqrt{36}$ ____

We have seen that the positive solution to $s^2 = 25$ is denoted $\sqrt{25}$. In general, the positive solution to $x^n = 25$ is $\sqrt[n]{25}$ and is called the **nth root of 25.** The number n is called the index. Note that in $\sqrt{25}$, the index 2 is understood and not expressed.

Sometimes the square root of a number is a rational number, sometimes it is an irrational number. The above examples resulted in rational numbers.

But $\sqrt{7}$ is not rational because there does not exist a rational number b such that $b^2 = 7$.

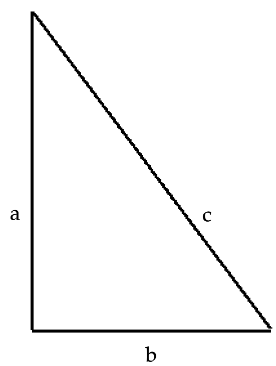
Whenever a cannot be written as q^2, where q is a rational number, then $\sqrt{a}$ is irrational.

Classify these numbers as rational or irrational:

$\sqrt{36}$ $\sqrt{24}$ $\sqrt{16}$ $\sqrt{8}$

Irrational Numbers are sometimes needed when using the Pythagorean Theorem.

Pythagorean Theorem - The square of the hypotenuse(c) of a right triangle is equal to the sum of the squares of the other two sides (a and b)



$$a^2 + b^2 = c^2$$

Example: Find c when a = 4 and b= 5 Find a when b = 6 and c = 7

The Set of Real Numbers is the union of Rational Numbers and Irrational Numbers.

REAL NUMBERS

Rationals **Irrationals**

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Integers

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Whole Numbers

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Natural Numbers

<u>More Properties of Exponents</u>:

a) $\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y}$

b) $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$

To simplify a radical, write the radicand in factored form, using the largest perfect square that works. If the radicand has no perfect square factor (other than 1), it is in <u>simplest radical form.</u>

$\sqrt{24}$

$\sqrt{27}$

$\sqrt{72}$

$\sqrt{128}$

$\sqrt{300}$

$\sqrt{125}$

$\sqrt[3]{54}$

$\sqrt[4]{48}$

$\sqrt[3]{32}$