

M119 – Practice Test 3

1) Evaluate using properties of logarithms (DO NOT USE A CALCULATOR):

a) $\ln\sqrt{e}$

b) $e^{2\ln 3}$

c) $e^{3\ln 2 - 2\ln 5}$

d) $\ln \frac{e^3\sqrt{e}}{e^{1/3}}$

Convert to a logarithmic equation.

2) a) $e^{-3} = \frac{1}{64}$

b) $5e^x = 2$

3) Convert from log form to exponential: a) $\ln x = 1.6094$

b) $\ln 39 = 3.6636$

Express as a single logarithm and simplify, if possible.

4) $\ln 42 - \ln 6$

5) $\ln x^2 - 2 \ln \sqrt{x}$

6) $\ln 17x + 2(\ln x - \ln y)$

7) $\ln w - 3[2\ln(w - 5) - \ln(w + 5)]$

8) $\frac{2}{3}[\ln(y^2 - 144) - \ln(y + 12)] + \ln(y + y)$

Solve the exponential equation algebraically.

9) a) $3x^2 + 5x = \frac{1}{81}$

b) $27 = 3^{5x} \cdot 9x^2$

10) a) $1000e^{4t} = 90$

b) $8 - 2e^{0.04x} = 0$

Solve the logarithm equation algebraically.

11) a) $\ln x = 17$

b) $\ln(6x - 1) = \ln 3 - \ln(x - 3)$

Differentiate.

12) a) $y = 4e^{x^2}$

b) $y = e^{3x^2} + x$

Find the derivative.

13) a) $y = 5x^2e^{3x}$

b) $y = (x^2 - 2x + 3)e^x$

14) a) $y = \frac{e^x}{3x^2 + 7}$

b) $y = (e^{x^3} - 1)^5$

Find the derivative of the function.

15) a) $y = \ln(7 + x^2)$

b) $y = \frac{\ln x}{x^5}$

Find the derivative.

16) a) $y = \frac{e^x}{\ln x}$

b) $f(x) = \ln(e^{4x} - 4)$

17) a) $f(x) = e^{8x} \ln x$

b) $y = \ln \sqrt{\frac{x^2 + 1}{2x + 3}}$

Differentiate.

18) $f(t) = \ln[(t^3 - 5)(t^6 + 1)]$

Find the indicated tangent line.

19) Find the tangent line to the graph of $f(x) = 2e^{4x}$ at the point (0, 2).

Solve.

- 20) How long will it take for the population of a certain country to triple if its annual growth rate is 1.1%? Assume the population is growing exponentially. Round to the nearest year.
- 21) Suppose that \$4000 is invested at an interest rate of 5.3% per year, compounded continuously. What is the balance after 3 years?
- 22) Suppose that \$9000 is invested at an interest rate of 5.4% per year, compounded continuously. What is the doubling time?

Solve the problem.

- 23) Ben Franklin bequeathed \$4000.00 to the city of Boston in 1790. Assuming the fund grew to \$6 million in 200 years,
- find the interest rate compounded continuously that would yield this total value.
 - How long did it take until the the balance reached \$1,000,000?
 - How fast was the balance changing when it reached \$1,000,000?

Solve.

- 24) A certain radioactive isotope has a half-life of approximately 1900 years. How many years to the nearest year would be required for a given amount of this isotope to decay to 80% of that amount?
- 25) An artifact is discovered at a certain site. If it has 57% of the carbon-14 it originally contained, what is the approximate age of the artifact to the nearest year? (carbon-14 decays at the rate of 0.0125% annually.)
- 26) A certain radioactive isotope decays at a rate of 0.3% annually. Determine the half-life of this isotope, to the nearest year.
- 27) The number of bacteria in a certain culture grows exponentially. If 5000 bacteria were present initially and 8,000 were present 10 minutes later, how long will it take for the number of bacteria to double?
- 28) The price of wheat per bushel at time t (in months) is $f(t) = 4 + .001t + .01e^{-t}$. What is the percentage rate of change at $t = 1$?

29) A lake is stocked with 100 fish. After 3 months there are 250 fish. The formula for the number of fish in the lake

$$t \text{ months after it is stocked is } P(t) = \frac{1000}{1 + Be^{-1000kt}}$$

- a) Find B.
- b) Find k
- c) How many fish will be in the lake after 4 months?

Solve the problem.

30) The demand function for a certain product is given by

$$D(p) = 800e^{-0.1p},$$

where p is price per unit. Recall that total revenue is given by $R(p) = pD(p)$. At what price per unit p will the revenue be maximum?

31) The aerobic rating of a person x years old is modeled by the function:

$$A(x) = \frac{110(-2 + \ln x)}{x} \quad \text{for } x \geq 10$$

At what age is a person's aerobic rating largest?

Graph

32) $y = 2xe^{-x}$

- fn is increasing _____
- fn is decreasing _____
- fn has a relative max at _____
- fn has a rel min at _____
- fn is concave up _____
- fn is concave down _____
- fn has an inflection point at _____

Answer Key

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- 1) a) $1/2$ b) 9 c) $8/25$ d) $19/6$
- 2) a) $\ln \frac{1}{64} = -3$ b) $x = \ln(2/5)$
- 3) a) $x = e^{1.6094}$ b) $e^{3.6636} = 39$
- 4) $\ln(7)$
- 5) $\ln x$
- 6) $\ln \frac{17x(3)}{y^2}$
- 7) $\ln \frac{w(w+5)^3}{(w-5)^6}$
- 8) $\ln (y-12)^{2/3}(y+y)$
- 9) a) $-1, -4$ b) $-3, 1/2$
- 10) a) $\ln .09/4$ b) $25 \ln 4$
- 11) a) $24,154,952.75$ b) $19/6$
- 12) a) $8xe^{x^2}$ b) $6xe^{3x^2} + 1$
- 13) a) $5xe^{3x}(3x+2)$ b) $(x^2+1)e^x$
- 14) a) $\frac{e^x(3x^2-6x+7)}{(3x^2+7)^2}$ b) $15x^2e^{x^3}(e^{x^3}-1)^4$
- 15) a) $\frac{2x}{x^2+7}$ b) $\frac{1-5\ln x}{x^6}$
- 16) a) $\frac{x e^x \ln x - e^x}{x \ln^2 x}$ b) $\frac{4e^{4x}}{e^{4x}-4}$
- 17) a) $\frac{e^{8x}(1+8x \ln x)}{x}$ b) $y' = \frac{x}{x^2+1} - \frac{1}{2x+3}$
- 18) $\frac{3t^2(t^6+1)+6t^5(t^3-5)}{(t^3-5)(t^6+1)}$
- 19) $y = 8x + 2$
- 20) 100 years
- 21) \$4689.35
- 22) 12.8 years
- 23) a) 3.7% b) about 149 years c) \$36,688.27/year
- 24) 612 years
- 25) 4497 years
- 26) 231 years
- 27) 14.75 minutes
- 28) -0.669%
- 29) a) $B = 9$ b) $k \approx .00037$ c) about 328 fish
- 30) \$10
- 31) when $x = 20.08$ or about 20 years old

Answer Key

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32) fn is increasing $(-\infty, 1)$

fn is decreasing $(1, \infty)$

fn has a relative max at $(1, 2/e)$

fn has a relative min at none

fn is concave up $(2, \infty)$

fn is concave down $(-\infty, 2)$

fn has an inflection point at $(2, 4/e^2)$

