M119 – Practice Test 2

Find the absolute maximum and absolute minimum (if they exist) for the following functions:

1) $f(x) = x^3 - 3x - 2$; $[-5, 1]$

Absolute max = ______________ Absolute min = _________________

2) $f(x) = x^4 - 32x^2$; $[-3, 6]$

Absolute max = ______________ Absolute min = _________________

3) If $y = x^3 - 12x + 2$, find

a) where the function is increasing ___________________________
b) where the function is decreasing ___________________________
c) where the function has a relative max _______________________
d) where the function has a relative min _______________________
e) where the function is concave up __________________________
f) where the function is concave down __________________________
g) where the function has inflection point(s) _____________________
h) graph the function

Differentiate.

4) $f(x) = (x^2 - 4x + 2)(3x^3 - x^2 + 5)$

5) $f(x) = (3x + 4)^2$

6) $f(x) = (4x - 4)(\sqrt{x} + 2)$

7) $g(x) = (x^{-5} + 3)(x^{-3} + 5)$

8) $g(x) = \dfrac{x^2}{x - 11}$

9) $y = \dfrac{x^2 - 3x + 2}{x^7 - 2}$

10) $y = \dfrac{x^2 + 8x + 3}{\sqrt{x}}$

11) $f(x) = \dfrac{(x + 4)(x + 2)}{(x - 4)(x - 2)}$

12) $f(x) = \dfrac{(x - 9)(x^2 + 3x)}{x^3}$

13) $f(x) = \left(\dfrac{3x^3 - 2x + 1}{x^2 - 2}\right)^3$ (Do not simplify)

Write an equation of the tangent line to the graph of y = f(x) at the point on the graph where x has the indicated value.

14) $f(x) = (-4x^2 - 5x - 2)(2x + 3)$, $x = 0$

15) $f(x) = \frac{-8x^2 - 5}{-4x - 1}$, $x = 0$

Find an expression for dy/dx.

16) $y = u(u - 1)$ and $u = x^2 + x$

17) $y = \frac{u + 2}{u - 2}$ and $u = \sqrt{x} + 5$

Calculate the requested derivative from the given information.

18) Given $f(u) = \sqrt[3]{u}$ and $g(x) = u = 1 + 2x^3$, find $(f \circ g)'(-1)$.

19) Given $f(u) = \frac{u - 1}{u + 1}$, $g(x) = u = \sqrt{x}$, find $(f \circ g)'(64)$.

Find $\frac{d^2y}{dx^2}$.

20) $y = (4x + 5)^3$

21) $y = (x^2 + 5x)^{40}$

Solve the problem.

22) Of all numbers whose sum is 500, find the two that have the maximum product. That is, maximize $Q = xy$, where $x + y = 500$.

23) A carpenter is building a rectangular room with a fixed perimeter of 460 ft. What are the dimensions of the largest room that can be built? What is its area?

24) From a thin piece of cardboard 30 in. by 30 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary.

25) A daily newspaper is currently charging subscribers $16 per month and has 42,000 subscribers. For every $1 increase in the monthly price, the paper will lose 3000 subscribers.
a) What should the newspaper charge per month to maximize its revenue?
b) What is the newspaper's maximum revenue?

Solve the problem.

26) A baseball team is trying to determine what price to charge for tickets. At a price of $10 per ticket, it averages 50,000 people per game. For every increase of $1, it loses 5,000 people. Every person at the game spends an average of $5 on concessions. What price per ticket should be charged in order to maximize revenue?

27) A company is constructing an open–top, square–based, rectangular metal tank that will have a volume of 25 ft^3. What dimensions yield the minimum surface area? Round to the nearest tenth, if necessary.

28) At a certain factory the total cost of manufacturing q units is $C(q) = 0.2q^2 + q + 900$ dollars. it has been determined that approximately $q(t) = t^2 + 100t$ units are manufactured during the first t hours of a production run. compute the rate at which the total manufacturing cost is changing with respect to time 1 hour after production.

Solve the problem.

29) Find the number of units that must be produced and sold in order to yield the maximum profit, given the following equations for revenue and cost:

$R(x) = 20x - 0.5x^2$

$C(x) = 8x + 7.$

30) A rectangular field is to be enclosed on four sides with a fence. Fencing costs $5 per foot for two opposite sides, and $8 per foot for the other two sides. Find the dimensions of the field of area 690 ft^2 that would be the cheapest to enclose.

31) (Remember **Revenue = x•p** and **Profit = Revenue – Cost**)

The demand equation for a certain fast food restaurant is given by $p = \frac{60,000 - x}{20,000}$, where x is the number of hamburgers and p is the price per unit. The cost of producing x hamburgers is

$$C = 5000 + 0.5x \qquad 0 \le x \le 50,000.$$

a) Use calculus to find the production level that produses a maximum profit.
b) Find the interval(s) where P is increasing.
c) Find the interval(s) where P is decreasing.
d) Find the corresponding price that will maximize the profit.
e) Find the maximum profit. _

32) Sketch a graph of a function that has all of the following properties:
a) $f'(0) = f'(1) = f'(2) = 0$
b) $f'(x)<)$ when $x<0$ and $x>2$
c) $f'(x)>0$ when $0<x<1$ and $1<x<2$

33) Sketch a graph of a function that has all of the following properties:
a) $f'(x) > 0$ when $x < -5$ and when $x > 1$
b) $f'(x) < 0$ when $-5 < x < 1$
c) $f(-5) = 4$ and $f(1) = -1$

Answer Key
Testname: M119–REVIEW_CH–2–3.TST

1) Absolute maximum: 0, absolute minimum: –112
2) Absolute maximum: 144, absolute minimum: –256
3) a) $x< -2$, $x>2$ b) $-2<x<2$ c) Local maximum at (–2, 18) d) local minimum at (2, –14) e) $x> 0$ f) $x< 0$ g) (0,2)
4) $f'(x) = 15x^4 - 52x^3 + 30x^2 + 6x - 20$
5) $f'(x) = 18x + 24$
6) $f'(x) = 6x^{1/2} - 2x^{-1/2} + 8$
7) $g'(x) = -8x^{-9} - 25x^{-6} - 9x^{-4}$
8) $g'(x) = \dfrac{x^2 - 22x}{(x - 11)^2}$
9) $y' = \dfrac{-5x^8 + 18x^7 - 14x^6 - 4x + 6}{(x^7 - 2)^2}$
10) $y' = \dfrac{3x^2 + 8x - 3}{2x^{3/2}}$
11) $f'(x) = \dfrac{-12x^2 + 96}{(x - 4)^2(x - 2)^2}$
12) $f'(x) = \dfrac{6}{x^2} + \dfrac{54}{x^3}$
13) $f'(x) = 3\left(\dfrac{3x^3-2x+1}{x^2-2}\right)^2 \dfrac{(x^2-2)(9x^2-2)-(3x^3-2x+1)(2x)}{(x^2-2)^2}$
14) $y = -19x - 6$
15) $y = - 20x + 5$
16) $4x^3 + 6x^2 - 1$
17) $\dfrac{-2}{\sqrt{x}(\sqrt{x} + 3)^2}$
18) 2
19) $\dfrac{1}{648}$
20) $384x + 480$
21) $40(x^2 + 5x)^{38}(158x^2 + 790x + 975)$
22) 250 and 250
23) 115 ft by 115 ft; 13,225 ft^2
24) 20 in. by 20 in. by 5 in.; 2000 $in.^3$
25) a) $15 b) $675,000
26) $7.50
27) 3.7 ft by 3.7 ft. by 1.8 ft
28) After 1 hour cost is changing by $4222.8/hour
29) 12 units
30) 33.2 ft @ $5 by 20.8 ft @ $8
31) a) 25,000 units b) $x < 25000$ c) $x > 25000$ d)$1.75
32) q
33) r