

M421, Introduction to Topology I

Assignment 3

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Theorem 1. *Let $f : X \rightarrow Y$ be a function and $A \subseteq Y$ and $B \subseteq Y$. Then $f^{-1}(A) - f^{-1}(B) = f^{-1}(A - B)$.*

Proof. Let $x \in X$. $x \in f^{-1}(A) - f^{-1}(B) \Leftrightarrow x \in f^{-1}(A), x \notin f^{-1}(B) \Leftrightarrow f(x) \in A, f(x) \notin B \Leftrightarrow f(x) \in A - B \Leftrightarrow x \in f^{-1}(A - B)$. \square

Pg. 32, number 8.

Use Definition 2.1.6 to show that

$$f(x) = \begin{cases} -3 & \text{if } x < 1 \\ 3 & \text{if } x \geq 1 \end{cases}$$

Proof. To prove this statement, let me restate the definition to be used.

Definition: A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is said to be continuous if for each open subset V of \mathbb{R} , $f^{-1}(V)$ is an open subset of \mathbb{R} .

Consider the open set $(-4, -2)$. $f^{-1}(-4, -2) = (-\infty, 1]$. \square