M421, Introduction to Topology I; Assignment 2

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Theorem 1. *Let A, B, C be sets. Then $A \times (B \cup C) = (A \times B) \cup (A \times C)$.*

Proof. Let (x, y) be an element. $(x, y) \in A \times (B \cup C) \Leftrightarrow x \in A$ and $y \in B \cup C \Leftrightarrow x \in A$ and $y \in B$ or $y \in c \Leftrightarrow x \in A$ and $y \in B$ or $x \in A$ and $y \in C \Leftrightarrow (x, y) \in (A \times B) \cup (A \times C)$. □

Theorem 2. *Let $\{A_\alpha : \alpha \in \Lambda\}$ be an indexed collection of subsets of X, then*

$$X - \bigcap_{\alpha \in \Lambda} A_\alpha = \bigcup_{\alpha \in \Lambda} (X - A_\alpha).$$

Proof. Let $x \in X$. $x \in X - \bigcap_\alpha A_\alpha \Leftrightarrow x \notin \bigcap_\alpha A_\alpha \Leftrightarrow \exists\ A_\alpha \ni: x \in A_\alpha \Leftrightarrow x \in \bigcup_\alpha X - A_\alpha$.
$\therefore X - \bigcap_\alpha A_\alpha = \bigcup_\alpha (X - A_\alpha)$. □

Theorem 3. *Let $\{A_\alpha : \alpha \in \Lambda\}$ be an indexed collection of sets, and let B be a set. Then $B \cup (\bigcap_\alpha A_\alpha) = \bigcap_\alpha (B \cup A_\alpha)$.*

Proof. $(\Rightarrow)$
Let x be an element. $x \in B \cup (\bigcap_\alpha A_\alpha) \Rightarrow x \in B$ or $x \in \bigcap_\alpha A_\alpha$. If $x \in B$, then $x \in B \cup A_\alpha \forall \alpha \Rightarrow x \in \bigcap_\alpha (B \cup A_\alpha)$. If $x \in \bigcap_\alpha A_\alpha$, then $x \in A_\alpha \forall \alpha \Rightarrow x \in B \cup A_\alpha \forall \alpha \Rightarrow \bigcap_\alpha B \cup A_\alpha$.
$(\Leftarrow)$
$x \in \bigcap_\alpha (B \cup A_\alpha)$, then $x \in B \cup A_\alpha \forall \alpha \Rightarrow x \in B$ or $x \in A_\alpha \forall \alpha$. If $x \in B$, then $x \in B \cup (\bigcap_\alpha A_\alpha)$. If $x \in A_\alpha \forall \alpha$, then $x \in \bigcap_\alpha A_\alpha \Rightarrow x \in B \cup (\bigcap_\alpha A_\alpha)$.
$\therefore B \cup (\bigcap_\alpha A_\alpha) = \bigcap_\alpha (B \cup A_\alpha)$. □