

M421, Introduction to Topology I

Assignment 11

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Theorem 1. *Let (X, \mathcal{T}) and (Y, \mathcal{S}) be spaces with $A \subseteq X$, (Y, \mathcal{S}) . Then*

$$\text{Int}(A \times B) = \text{Int}(A) \times \text{Int}(B).$$

Proof. (\subseteq)

Let $(x, y) \in \text{Int}(A \times B)$, then $\exists U \subseteq A, V \subseteq B$ $\ni U \times V \subseteq A \times B$ and $(x, y) \in U \times V \subseteq A \times B$. Then $x \in U \subseteq A, y \in V \subseteq B \Rightarrow x \in \text{Int}(A), y \in \text{Int}(B)$.

$\therefore (x, y) \in \text{Int}(A) \times \text{Int}(B)$. □

Proof. (\supseteq)

Let $(x, y) \in \text{Int}(A) \times \text{Int}(B)$, then $\exists U \subseteq A, V \subseteq B$ s.t. $x \in U, y \in V$. Thus $(x, y) \in U \times V \Rightarrow (x, y) \in \text{Int}(A \times B)$. □