M421, Introduction to Topology I
Assignment 9

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Theorem 1. *Let $\mathscr{S}$ and $\mathscr{T}$ be topologies for a set X. Then the identity function*

$$I : (X, \mathscr{T}) \to (X, \mathscr{S})$$

is an open function $\Leftrightarrow$ $\mathscr{S}$ is finer than $\mathscr{T}$ (that is $\mathscr{T} \subseteq \mathscr{S}$.)

Proof. Proof. $(\Rightarrow)$
Let $U \in \mathscr{T}$. Then because I is open, $I(U) = U \in \mathscr{S}$. Thus $\mathscr{T} \subseteq \mathscr{S}$. □

Proof. $(\Leftarrow)$
Suppose $\mathscr{T} \subseteq \mathscr{S}$. Then $U \in \mathscr{T} \Rightarrow U \in \mathscr{S}$. Since $U = I(U)$, $I(U) \in \mathscr{S}$.
$\therefore$ I is open. □

□

Theorem 2. *Let $(X, \mathscr{T})$, $(Y, \mathscr{S})$, and $(Y, \mathscr{F})$ be topological spaces and let $f : X \to Y$ be a function. If f is $\mathscr{T} - \mathscr{F}$ continuous and $\mathscr{S}$ is finer that $\mathscr{F}$, then f is $\mathscr{T} - \mathscr{S}$ continuous.*

Proof. Let $V \in \mathscr{F}$. Then because $\mathscr{F} \subseteq \mathscr{S}$, $V \in \mathscr{S}$. And since $f^{-1}(V) \in \mathscr{T}$, $f : (X, \mathscr{T}) \to (Y, \mathscr{S})$ is $\mathscr{T} - \mathscr{S}$. □

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Write the letters of the alphabet as block capital letters. Partition these "spaces" into mutually dijoint collections of homeomorphic spaces such that any two spaces from different collections are not homeomorphic.