M421, Introduction to Topology I; Assignment 1

Drew Robertson

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Section 1.2

Theorem 1. *Let X be a set, and $A, B \subset X$. Then $X \backslash (A \cup B) = (X \backslash A) \cap (X \backslash B)$.*

Proof. ($\subset$)
See notes.

($\supset$)
To prove this direction, we must show that $(X \backslash A) \cap (X \backslash B) \subset X - (A \cup B)$. To start, let $x \in (X \backslash A) \cap (X \backslash B)$, that is $x \in X \backslash A$ and $x \in X \backslash B$. This means that $x \notin A$ or $x \notin B$. WOLG, if $x \notin A$, then $x \in B \Rightarrow x \in X - (A \cup B)$.
$\therefore (X \backslash A) \cap (X \backslash B) \subset X - (A \cup B)$.
$\therefore$ Since containment is shown in both directions, $(X \backslash A) \cap (X \backslash B) = X \backslash (A \cup B)$. □

Theorem 2. *If A, B, C are sets, then*

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

Proof. ($\subset$)
Let $x \in A \cap (B \cup C)$. Then, $x \in A$ and $x \in B \cup C$, that is $x \in A$ and $x \in B$ or $x \in C$. So now we have two cases, first case is $x \in B$, the second case is $x \in C$.
Assume case 1. Then $x \in A \cap B \Rightarrow x \in (A \cap B) \cup (A \cap C)$. Case 2 holds by similar logic.
$\therefore A \cap (B \cup C) \subset (A \cap B) \cup (A \cap C)$.

($\supset$)
Let $x \in (A \cap B) \cup (A \cap C)$. Then $x \in A \cap B$ or $x \in A \cap C \Rightarrow x \in A$ and B or $x \in A$ and $C \Rightarrow x \in A$ and $x \in B$ or $C \Rightarrow x \in A \cap (B \cup C)$.
□