

M360  
Elements of Probability; Assignment 8

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October 16, 2008

**Question 1.** *Sketch the graph of the following density functions. Also find and sketch the graphs of the associated distribution functions.*

(a)  $f(x) = 3x^2/2$

(b)  $f(x) = 1/2$

(c)  $f(x) = \begin{cases} x + 1, & x \in (-1, 0) \\ 1 - x, & x \in [0, 1) \end{cases}$

*Solution:*

(a)  $F(x) = \begin{cases} 0, & x \leq -1 \\ \frac{x^3}{2}, & x \in (-1, 1) \\ 1, & x \geq 1 \end{cases}$

(b)  $F(x) = \begin{cases} 0, & x \leq -1 \\ \frac{x}{2}, & x \in (-1, 1) \\ 1, & x \geq 1 \end{cases}$

(c)  $F(x) = \begin{cases} 0, & x \leq -1 \\ \frac{x^2}{2} + x + \frac{1}{2}, & x \in (-1, 0) \\ x - \frac{x^2}{2}, & x \in (0, 1] \\ 1, & x \geq 1 \end{cases}$

□

**Question 2.** Find  $\mu$  and  $\sigma^2$  of the p.d.f.'s in the previous problem.

Solution: (a)

$$\mu = 3/2 \int_{-1}^1 x^3 dx = 3/8 - 3/8 = 0. \quad \sigma^2 = 3/2 \int_{-1}^1 x^4 dx = 3x^5/10 \Big|_{-1}^1 = 3/5.$$

$$(b) \quad \mu = \int_{-1}^1 1/2 dx = 0. \quad \sigma^2 = \int_{-1}^1 x^2/2 dx = 1/3$$

$$(c) \quad \mu = \int_{-1}^0 (x+1) dx + \int_0^1 (1-x) dx = 0. \quad \sigma^2 = \int_{-1}^0 x^2(x+1) dx + \int_0^1 x^2(1-x) dx = 1/6$$

□