M360
Elements of Probability; Assignment 6

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Section 2.4, numbers 14 and 17. Section 2.5, numbers 2, 6, 18

Question 1. *A certain type of mint has a label weight of* $20.4g$*. Suppose that the probability is* 0.90 *that a mint weighs more than* $20.7g$*. Let* X *equal the number of mints that weight more than* $20.7g$ *in a sample of* 8 *mints selected at random.*
(a) How is X *distributed if we assume independence?*
(b) Find
(i) $P(X = 8)$*, (ii)* $P(X \leq 6)$*, (iii)* $P(X \geq 6)$

Solution:
Note that $f(x) = \binom{8}{x} 0.9^x \cdot 0.1^{8-x}$, for $x \in \{1, 2, ..., 8\}$.
(a)

$$\sum_{x=0}^{8} \binom{8}{x} \cdot 0.9^x \cdot 0.1^{8-x}$$

(b)
(i)

$$P(X = 8) = f(8) = 0.9^8 = 0.43046721$$

(ii)

$$P(X \leq 6) = f(X \leq 6) = \sum_{x=0}^{6} \binom{8}{x} \cdot 0.9^x \cdot 0.1^{8-x} = 0.187$$

(iii)

$$P(X \geq 6) = f(X \geq 6) = \sum_{x=6}^{8} \binom{8}{x} \cdot 0.9^x \cdot 0.1^{8-x} = 0.962$$

□

Question 2. *It is claimed that for a particular lottery,* $1/10$ *of the* $50,000,000$ *tickets will win a prize. What is the probability of winning at least one prize if you purchase*
(a) 10 tickets or
(b) 15 tickets?

Solution:
(a) $f(0) = (9/10)^{10} \Rightarrow f(X \neq 0) = 1 - (9/10)^{10} \approx 0.3487$
(b) $f(0) = (9/10)^{15} \Rightarrow f(X \neq 0) = 1 - (9/10)^{15} \approx 0.2059$

□

Question 3. *(i) Give the name of the distribution of X (if it has a name);*
(ii) Find the values of μ and σ^2;
(iii) Calculate $P(1 \leq X \leq 2)$ when the moment generating function is given by
(a) $M(t) = (0.3 + 0.7e^t)^5$
(b) $M(t) = \frac{0.3e^t}{1-0.7e^t}$
(c) $M(t) = 0.45 + 0.55e^t$
(d) $M(t) = 0.3e^t + 0.4e^{2t} + 0.2e^{3t} + 0.1e^{4t}$
(e) $M(t) = (0.6e^t)^2(1 - 0.4e^t)^{-2}$
(f) $M(t) = \sum_{x=1}^{1} 0(0.1)e^{tx}$

Solution:
(a) $M(t) = (0.3 + 0.7e^t)^5$ is a binomial distribution. $M'(t) = 5\left(0.3 + 0.7e^t\right)^4 \cdot 0.7e^t$, and $\mu = M'(0) = 5(0.3 + 0.7)^4 = 5$ $M''(t) = 20\left(0.3 + 0.7e^t\right)^3$, then $\sigma^2 = M''(0) - \mu^2 = 20 - 5 = 15$
(b) $M(t) = \frac{0.3e^t}{1-0.7e^t}$ is a geometric distribution. $\mu = 1/0.3$, $\sigma^2 = 0.7/0.3^2$.
(c) $M(t) = 0.45 + 0.55e^t$, is Bernouli, then $\mu = 0.55$, $\sigma^2 = 0.2475$
(d) $M(t) = 0.3e^t+0.4e^{2t}+0.2e^{3t}+0.1e^{4t}$ is not a usual distribution, then $\mu = M'(t) = 0.3+0.8+0.6+0.4 = 2.1$, $\sigma^2 = 5.09$
(e) $M(t) = (0.6e^t)^2(1 - 0.4e^t)^{-2}$ is a negative binomial distribution, so $\mu = -2(1/.6) = -3.3333$, $\sigma^2 = -2(.4)/.6^2 = -2.2222$
(f) $M(t) = \sum_{x=1}^{1} 0(0.1)e^{tx}$ is a uniform distribution, so $\mu = 11/2 = 5.5$, $\sigma^2 = 99/12 = 8.25$

□

Question 4. *The probability that a machine produces a defective item is* 0.01*. Each item is checked as it is produced. Assume that these are indpendent trials and compute checked as produced. Assume that these are independent trials and compute the probability that at least 100 items must be checked to find one that is defective.*

Solution: We note that this is a geometric distribution. So $f(x) = (1-p)^{x-1}p, x \in \mathbb{N}$. Note now that $p = 0.01$ and we wish to find $f(x \geq 100) = 1 - f(x < 100)$ So, we have

$$f(x \geq 100) = 1 - f(x < 100) = 1 - \sum_{x=1}^{99}(1 - 0.01)^{x-1} \cdot 0.01 = 0.3697$$

□