M360
Elements of Probability; Assignment 4

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Section 2.1, # 3,11

Question 1. *For each of the following, determine the constant c so that $f(x)$ satisfies the conditions for being a p.m.f. for a random variable X. Depict each p.m.f as a bar graph.*

Solution: (a) $f(x) = x/c$, $x = 1, 2, 3, 4$
$\sum f(x) = 1 \Rightarrow c = \sum_{x=1}^{4} x = 10$

(b) $f(x) = cx$, $x = 1, 2, ..., 10$
$1 = \sum_{x=1}^{10} cx \Rightarrow c = 1/\left(\sum_{x=1}^{10} x\right) = 1/55$

(c) $f(x) = c(1/4)^x$, $x = 1, 2, 3, ...$
$c = 1/\left(\sum_{x \in \mathbb{N}} (1/4)^x\right) = 3$

(d) $f(x) = c(x + 1)^2$, $x = 0, 1, 2, 3$
$c = 1/\sum_{x=0}^{3} (x + 1)^2 = 30$

(e) $f(x) = x/c$, $x = 1, 2, ..., n$
$c = \sum_{x=1}^{n} x = n(n + 1)/2$

□

Question 2. *In a lot of 100 light bulbs, there are* 5 *bad bulbs. An inspector inspects 10 bulbs selected at random. Find the probability of finding at least one defective bulb.*

Solution: Let $X =$ the number of bad bulbs selected. The

$$P(X = 0) = \frac{\binom{5}{0}\binom{95}{10}}{\binom{100}{10}} \approx 0.58375 \qquad \text{Thus}$$
$$P(X \neq 0) = 1 - 0.58375 = 0.41625$$

□