M360
Elements of Probability; Assignment 3

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Section 1.5, numbers 7,11 & Section 1.6, numbers 4,7

Question 1. *Each of three football players attempt to kick a field goal from the* 25*-yard line. Let* A_i *denote the event that the field goal is made by player* $i \in \{1, 2, 3\}$*. Assume that* A_1, A_2, A_3 *are mutually independent and that* $P(A_1) = 0.5$*,* $P(A_2) = 0.7$*, and* $P(3) = 0.6$*.*
(a.) Compute the probability that exactly one player is successful.
(b.) Compute the probability that exactly two players make a field goal.

Solution: (a.) First, by definition, $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$ and $P(A_1) \cap P(A_2) = P(A_1)P(A_2), P(A_1) \cap P(A_3) = P(A_1)P(A_3), P(A_2) \cap P(A_3) = P(A_2)P(A_3)$. Note now that the probability of one being successful is (by theorem completed in class)

$$P(\text{one successful}) = P(A_1 \cap A_2' \cap A_3') + P(A_2 \cap A_1' \cap A_3') + P(A_3 \cap A_2' \cap A_1')$$
$$= P(A_1)P(A_2')P(A_3') + P(A_2)P(A_1')P(A_3') + P(A_3)P(A_2')P(A_1').$$
$$= 0.06 + 0.14 + 0.09 = 0.29$$

(b.) For this problem, we use similar logic to the above.

$$P(\text{one unsuccessful}) = P(A_1' \cap A_2 \cap A_3) + P(A_2' \cap A_1 \cap A_3) + P(A_3' \cap A_1 \cap A_2)$$
$$= P(A_1')P(A_2)P(A_3) + P(A_2')P(A_1)P(A_3) + P(A_3') + P(A_1)P(A_2)$$
$$= 0.21 + 0.09 + 0.14 = 0.44$$

□

Question 2. *Let* A *and* B *be two events.*
(a.) If the events A, B *are mutually exclusive, are* A *and* B *always indpendent?*
(b.) If $A \subset B$*, can* A *and* B*ever be independent events?*

Solution: (a.) No. Suppose A, B are independent, then $P(A \cap B) = 0 \Rightarrow P(A) = 0$ or $P(B) = 0$. Thus if $P(A), P(B) \neq 0$, then $P(A)$ and $P(B)$ are dependent.

(b.) No. Since $A \subset B \Rightarrow P(A \cap B) = P(A)$. Thus if $P(B) \neq 1$, this can't happen.

□

Question 3. *Assume that an insurance company knows the below probabilities relating to automobile accidents. A randomly selected drive from the company's insured drivers has an accident. What is the conditional probability that the driver is in the* $16-25$ *age group?*

Partition Number	Age of driver	Probability of acident	Fraction of Company's Insured Drivers
I	16-25	0.005	0.10
II	26-50	0.02	0.55
III	51-65	0.03	0.20
IV	66-90	0.04	0.15

Solution: Let A be the event of having an accident. Then $P(A) = P(AI) + P(AII) + P(AIII) + P(AIV) = P(I)P(A|I) + P(II)P(A|II) + P(III)P(A|III) + P(IV)P(A|IV) = 0.1(0.005) + 0.55(0.02) + 0.20(0.03) + 0.15(0.04) = 0.0181$. So

$$P(I|A) = \frac{0.1 \cdot 0.005}{0.0181} = 0.02762.$$

□

Question 4. *Among 60 year old college professors,* 10% *are smokeres and* 90% *are non-smokers. The probability of a non-smoker dying next year is* 0.005 *and the probability of a smoker dying next year is* 0.05*. Given that one of this group of college professors dies in the next year, what is the conditional probability that the professor that dies is a smoker?*

Solution: Let $\mathscr{S} = \{\text{of 60 yr old professors}\}$. Then $\mathscr{S}$ can be partitioned into smokers, S, and non-smokers, N. Now, smokers S, make up 0.10 of the entire sample space, and non-smokers N make up 0.90 of the entire sample space $\mathscr{S}$. Let D be the event of death. Then $P(D|N) = 0.005$, $P(D|N) = 0.05$, so the $P(D) = 0.005 + 0.05 = 0.0095$. By Bayes's theorem, $P(N|D) = (0.0045)/(0.0095) = 0.47368$

□