

M360
Elements of Probability; Assignment 2

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Question 1. *In a string of Christmas tree light bulbs, 3 are defective. The bulbs are selected at random and tested, one at a time, until the defective bulb is found. Compute the probability that the third bulb is the (a.) third bulb tested, (b.) fifth bulb tested, and (c.) the tenth bulb tested.*

Solution: (a.) $P = \frac{1}{\binom{12}{3}}$

□

Solution: (b.) If we assume that the third bad bulb is in the fifth slot tested, then there are $\binom{4}{2}$ places for the other two bad bulbs to be. So, the probability of the third bulb being bad is

$$P = \frac{\binom{4}{2}}{\binom{52}{5}}.$$

□

Solution: (c.) By the similar logic as (b.), the probability that the 10th bulb is bad is

$$P \frac{\binom{9}{2}}{13}.$$

□

Question 2. *Bowl A contains three red and two white chips, and bowl B contains four red and three white chips. A chip is drawn at random from bowl A and transferred to bowl B. Compute the probability of then drawing a red chip from B.*

Solution: Let $S = \{E_i : i \in \{1, 2, 3, 4\}\}$ where $E_1 =$ red out of A then red out of B, $E_2 =$ white out of A with red out of B, $E_3 =$ white out of A and white of B, and $E_4 =$ red out of A with white out of B.

$$P(E_1) = 3/5 \cdot 5/8 = 15/40$$

$$P(E_2) = 2/5 \cdot 4/8 = 8/40$$

$$P(E_3) = 2/5 \cdot 4/8 = 8/40$$

$$P(E_4) = 3/5 \cdot 3/8 = 9/40$$

Note that $\sum_i P(E_i) = 1$, and $\bigcap_i E_i = \emptyset$. Thus we can say that $P(\text{red drawn from B}) = P(E_1) + P(E_2) = (15 + 8)/40 = 23/40$

□