

M413  
Introduction to Analysis I  
Assignment XV and Assignment XVI

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**Theorem 1.** *If  $(s_n)$  is unbounded non-increasing sequence, then  $\lim(s_n) = -\infty$ .*

*Proof.* Let  $(s_n)$  be an un-bounded non-increasing sequence. Let  $M < 0$ . Since  $\{s_n : n \in \mathbb{N}\}$  is unbounded and is bounded from above by  $s_1$ , it must be unbounded below. Thus, for some  $N \in \mathbb{N}$ ,  $s_N < M$ . Clearly,  $n > N \Rightarrow s_n < s_N < M$ , so  $\lim s_n = -\infty$ . □

**Question 2.** *Let  $a_n = 3 + 2(-1)^n, \forall n \in \mathbb{N}$ .*

*(a) List the first eight terms of  $(a_n)$*

*(b) Give a subsequence that is constant [takes a single value]. Specify the selection function  $\sigma$ .*

*Solution:* (a)  $(a_n)_{n \in \{1, 2, \dots, 8\}} = 1, 5, 1, 5, 1, 5, 1, 5$   
(b)  $b_k = a_{\sigma(k)} = 3 + 2(-1)^{2k} = 5$ . That is,  $\sigma(k) = 2k, \forall k \in \mathbb{N}$ .

□