M413
Introduction to Analysis I
Assignment XIV

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Problem 1. *Find* $\limsup(s_n)$ *and* $\liminf(s_n)$ *for* $s_n =$
(a) $1 + 1/(2n)$
(b) $1 - 1/n$
(c) $\sin(n\pi/4)$
(d) n^2

Solution: (a)
$\limsup(s_n) = \lim_{N\to\infty} \sup\{1 + 1/(2N+1), 1 + 1/(2n+2), ...\} = \lim_{N\to\infty}(1 + 1/(2N+1) = 1.$
$\liminf(s_n) = \lim_{N\to\infty} \sup\{1 + 1/(2N+1), 1 + 1/(2n+2), ...\} = 1.$

□

Solution: (b)
$\limsup(s_n) = \lim_{N\to\infty} \sup\{1 - 1/(N+1), 1 - 1/(N-2), ...\} = 1$
$\liminf(s_n) = \lim_{N\to\infty} \inf\{1 - 1/(N+1), 1 - 1/(N-2), ...\} = \lim_{N\to\infty}(1 - 1/(N+1) = 1.$

□

Solution: (c)
$\limsup(\sin(n\pi/4)) = \limsup\{-1, -\sqrt{2}/2, 0, \sqrt{2}/2, 1\} = 1$
$\liminf(s_n) = \liminf(\sin(n\pi/4)) = \liminf\{-1, -\sqrt{2}/2, 0, \sqrt{2}/2, 1\} = -1$

□

Solution: (d)
$\limsup(s_n) = \lim_{N\to\infty} \sup(n^2) = \lim_{N\to\infty} \sup\{(N+1)^2, ...\} = \infty.$
$\liminf(s_n) = \lim_{N\to\infty} \inf(n^2) = \lim_{N\to\infty} \inf\{(N+1)^2, ...\} = \infty.$

□