M413
Introduction to Analysis I
Assignment XIII

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Problem 1. *Prove* $\lim \frac{n^2+4}{n+5} = \infty$.

Discussion 1. $\forall M > 0$, we want $N \ni$: $n > N \Rightarrow \frac{n^2+4}{n+5} > M$. That is, we want $\frac{n^2+4}{n+5}$ >something. Note $\frac{n^2+4}{n+5} > n^2/6n = n/6$. We want $n/6 > M \Rightarrow n > 6M = N$.

Proof. Let $M > 0$. Let $N = 6M$, and let $n > N$. Thus $n > 6M$, and hence $n/6 > M$. Then

$$\frac{n^2+4}{n+5} > \frac{n}{6} > M.$$
$$\therefore \lim \frac{n^2+4}{n+5} = \infty$$

□

Problem 2. *Prove* $\lim \frac{6n^3+2}{3n+4} = \infty$.

Discussion 2. $\forall M > 0$, we want $N \ni$: $n > N \Rightarrow \frac{6n^3+2}{3n+4} > M$. Note that $\frac{6n^3+2}{3n+4} > \frac{6n^3}{7n} = \frac{6n^2}{7} > M \Rightarrow n > \sqrt{\frac{7M}{6}}$.

Proof. Let $M > 0$. Let $N = \sqrt{\frac{7M}{6}}$. Let $n > N$, then $n\sqrt{\frac{7M}{6}}$. Now

$$\frac{6n^3+2}{3n+4} > \frac{6n^2}{7} > M.$$
$$\therefore \lim \frac{6n^3+2}{3n+4} = \infty$$

□

Problem 3. *Suppose* $\exists N_0 \ni: s_n \leq t_n \forall n > N_0$.
(a) Prove that if $\lim s_n = \infty$, *then* $\lim t_n = \infty$.
(b) Prove that if $\lim t_n = -\infty$, *then* $\lim s_n = -\infty$.

Proof. (a)
Let $M > 0$. Since $\lim s_n = \infty, \exists N \ni: n > N \Rightarrow s_n > M$. Choose $N = N_0$, and let $n > N_0$. Then $t_n \geq s_n > M$, that is $t_n > M$.
$\therefore \lim t_n = \infty$. □

Proof. (b)
Since $\lim t_n = -\infty$, $\forall M < 0$, $\exists N \ni: n > N \Rightarrow t_n < M$. Choose $N = N_0$. Now by assumption, $s_n \leq t_n$ and by the definition of a limit going to negative infinity, $t_n < M$
$\therefore s_N < M$.
$\therefore s_n = -\infty$. □

Problem 4. *Show that if* $\lim s_n = \infty$ *and* $k > 0$, *then* $\lim ks_n = \infty$.

Discussion 3. $\forall M > 0$, we want $N \ni: n > N \Rightarrow ks_n > M$. That is we want $s_N > M/k$.

Proof. Let $M, k > 0$. Since $\lim s_n = \infty, \exists N \ni: n > N \Rightarrow s_n > M/k$. Let $n > N$. Since $s_n > M/k$ and $k > 0$, $ks_n > M$. Therefore, if $\lim s_n = \infty$, and $k > 0$, $\lim ks_n = \infty$. □