

M413
Introduction to Analysis I
Assignment XII

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Problem 1. *Suppose that $\lim a_n = a$, $\lim b_n = b$, and that*

$$s_n = \frac{a_n^3 + 4a_n}{b_n^2 + 1}.$$

Prove that $\lim s_n = \frac{a^3 + 4a}{b^2 + 1}$.

Proof. $\lim a_n^3 = a^3$, by the product theorem, $\lim 4a_n = 4a$ by the constant multiple theorem. Similarly, $\lim b_n^2 = b^2$ by the constant multiple rule and $\lim 1 = 1$ by the constant theorem. Thus $\lim a_n^3 + 4a_n = a^3 + 4a$ and $\lim b_n^2 + 1 = b^2 + 1$ by the sum theorem. Thus by the quotient theorem, $\lim s_n = \frac{a^3 + 4a}{b^2 + 1}$. \square