M413
Introduction to Analysis I
Assignment VI

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Question 1. *Find* $\sup(S)$ *and* $\inf(S)$ *for* $S =$
(e) $\{1/n : n \in \mathbb{N}\}$
(f) $\{0\}$
(g) $[0, 1] \cup [2, 3]$
(h) $\bigcup_{n\in\mathbb{N}}[2n, 2n + 1]$
(i) $\bigcap_{n\in\mathbb{N}}[-1/n, 1 + 1/n]$
(j) $\{1 - 1/3^n : n \in \mathbb{N}\}$
(k) $\{n + (-1)^n/n : n \in N\}$
(l) $\{r \in \mathbb{Q} : r < 2\}$
(m) $\{r \in \mathbb{Q} : r^2 < 4\}$
(n) $\{r \in \mathbb{Q} : r^2 < 2\}$
(o) $\{x \in \mathbb{R} : x < 0\}$
(p) $\{1, \pi/3, \pi^2, 10\}$
(q) $\{0, 1, 2, 4, 8, 16\}$
(r) $\bigcap_{n\in\mathbb{N}}(1 - 1/n, 1 + 1/n)$
(s) $\{1/n : n \in \mathbb{N}$ *and n is prime*$\}$
(t) $\{x \in \mathbb{R} : x^3 < 8\}$
(u) $\{x^2 : x \in \mathbb{R}\}$
(v) $\{\cos(n\pi/3) : n \in \mathbb{N}\}$
(w) $\{\sin(n\pi/3) : n \in \mathbb{N}\}$.

Solution: (e) $\sup(\{1/n : n \in \mathbb{N}\}) = 1$, $\inf(\{1/n : n \in \mathbb{N}\}) = 0$
(f) $\sup(\{0\} = 0 = \inf(\{0\})$
(g) $\sup([0, 1] \cup [2, 3]) = 3$, $\inf([0, 1] \cup [2, 3]) = 0$
(h) $\sup(\bigcup_{n\in\mathbb{N}}[2n, 2n + 1]) = DNE$, $\inf(\bigcup_{n\in\mathbb{N}}[2n, 2n + 1]) = 2$
(i) $\sup(\bigcap_{n\in\mathbb{N}}[-1/n, 1 + 1/n]) = 1$, $\inf(\bigcap_{n\in\mathbb{N}}[-1/n, 1 + 1/n]) = 0$
(j) $\sup(\{1 - 1/3^n : n \in \mathbb{N}\}v) = 1$, $\inf(\{1 - 1/3^n : n \in \mathbb{N}\}) = 2/3$
(k) $\sup(\{n + (-1)^n/n : n \in N\}) = 1$, $\inf(\{n + (-1)^n/n : n \in N\}) = 0$
(l) $\sup(\{r \in \mathbb{Q} : r < 2\}) = 2$, $\inf(\{r \in \mathbb{Q} : r < 2\}) = DNE$
(m) $\sup(\{r \in \mathbb{Q} : r^2 < 4\}) = 2$, $\inf(\{r \in \mathbb{Q} : r^2 < 4\}) = -2$
(n) $\sup(\{r \in \mathbb{Q} : r^2 < 2\}) = \sqrt{2}$, $\inf(\{r \in \mathbb{Q} : r^2 < 2\}) = -\sqrt{2}$
(o) $\sup(\{x \in \mathbb{R} : x < 0\}) = 0$, $\inf(\{x \in \mathbb{R} : x < 0\}) = DNE$
(p) $\sup(\{1, \pi/3, \pi^2, 10\}) = 10$, $\inf(\{1, \pi/3, \pi^2, 10\}) = 1$
(q) $\sup(\{0, 1, 2, 4, 8, 16\}) = 16$, $\inf(\{0, 1, 2, 4, 8, 16\}) = 0$
(r) $\sup(\bigcap_{n\in\mathbb{N}}(1 - 1/n, 1 + 1/n)) = \inf(\bigcap_{n\in\mathbb{N}}(1 - 1/n, 1 + 1/n)) = 1$
(s) $\sup(\{1/n : n \in \mathbb{N}) = 1$, $\inf(\{1/n : n \in \mathbb{N}) = 0$
(t) $\sup(\{x \in \mathbb{R} : x^3 < 8\}) = 2$, $\inf(\{x \in \mathbb{R} : x^3 < 8\}) = DNE$

(u) $\sup(\{x^2 : x \in \mathbb{R}\}) = DNE$, $\inf(\{x^2 : x \in \mathbb{R}\}) = 0$
(v) $\sup(\{\cos(n\pi/3) : n \in \mathbb{N}\}) = 1$, $\inf(\{\cos(n\pi/3) : n \in \mathbb{N}\}) = -1$
(w) $\sup(\{\sin(n\pi/3) : n \in \mathbb{N}\}) = \sqrt{2}/2$, $\inf(\{\sin(n\pi/3) : n \in \mathbb{N}\}) = -\sqrt{3}/2$

□

Theorem 2. *Let S, T be non-empty subsets of $\mathbb{R}$ that are bounded from below. Then if $S \subset T$, then $\inf(T) \leq \inf(S)$.*

Proof. Since $S \subset T$, any lower bound of T is a lower bound of S. Thus $\forall s \in S$, $\inf(T) \leq s$, that is $\inf(T)$ is a lower bound for S. $\inf(S)$ is the greatest lower bound of S, therefore $\inf(T) \leq \inf(S)$.

□