M413
Introduction to Analysis I
Assignment V

Drew Robertson

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Section 4, numbers $4.1 - 4.3$*, on* 4.3 *only through part d.*

Question 1. *(4.1-4.2) For each set below that is bounded above, list three upper bounds (lower bounds) for the set. Otherwise write NBA (NBB).*

(a) $[0,1]$
(b) $(0,1)$
(c) $\{2,7\}$
(d) $\{\pi, e\}$
(e) $\{1/n : n \in \mathbb{N}\}$
(f) $\{0\}$
(g) $[0,1] \cup [2,3]$
(h) $\bigcup_{n\in\mathbb{N}}[2n, 2n+1]$
(i) $\bigcap_{n\in\mathbb{N}}[-1/n, 1+1/n]$
(j) $\{1 - 1/3^n : n \in \mathbb{N}\}$
(k) $\{n + (-1)^n/n : n \in N\}$
(l) $\{r \in \mathbb{Q} : r < 2\}$
(m) $\{r \in \mathbb{Q} : r^2 < 4\}$
(n) $\{r \in \mathbb{Q} : r^2 < 2\}$
(o) $\{x \in \mathbb{R} : x < 0\}$
(p) $\{1, \pi/3, \pi^2, 10\}$
(q) $\{0, 1, 2, 4, 8, 16\}$
(r) $\bigcap_{n\in\mathbb{N}}(1 - 1/n, 1 + 1/n)$
(s) $\{1/n : n \in \mathbb{N}$ *and* n *is prime*$\}$
(t) $\{x \in \mathbb{R} : x^3 < 8\}$
(u) $\{x^2 : x \in \mathbb{R}\}$
(v) $\{\cos(n\pi/3) : n \in \mathbb{N}\}$
(w) $\{\sin(n\pi/3) : n \in \mathbb{N}\}$.

Solution: (a) The set is denoted in simplest terms already. Upper bounds will be $1, 2, 3$; lower bounds will be $0, -1, -2$.
(b) Same solutions as (a).
(c) This set is also defined in simplest terms already. Upper bounds will be $7, 8, 9$ and lower bounds will be $2, 1, 0$.
(d) This set is in simplest terms. Upper bounds are $\pi, 4, 5$ and lower bounds are $e, 1, 0$.
(e) This set is in simplest terms. Upper bounds are $1, 2, 3$, and lower bounds are $0, -1, -2$.
(f) This set is in simplest terms. Upper bounds are $0, 1, 2$ and lower bounds are $0, -1, -2$.
(g) This set is in simplest terms. Upper bounds are $3, 4, 5$ and lower bounds are $0, -1, -2$.
(h) $\bigcup_{n\in\mathbb{N}}[2n, 2n+1] = [2, \infty]$. This set is NBA, and has lower bounds $2, 1, 0$.
(i) $\bigcap_{n\in\mathbb{N}}[-1/n, 1+1/n] = (0,1)$ has upper bounds $1, 2, 3$ and lower bounds $0, -1, -2$.
(j) $\{1 - 1/3^n : n \in \mathbb{N}\}$ is bounded from above by $1, 2, 3$ and lower bounds are $2/3, 1/3, 0$.
(k) $\{n + (-1)^n/n : n \in \mathbb{N}\}$ is bounded from below by $0, -1, -2$ and from above by $1, 2, 3$.
(l) $\{r \in \mathbb{Q} : r < 2\}$ is NBB but has upper bounds of $2, 3, 4$.
(m) $\{r \in \mathbb{Q} : r^2 < 4\} = \{r \in \mathbb{Q} : -2 < r < 2\}$, and thus is bounded from above by $2, 3, 4$ and from below by $-2, -3, -4$.

(n) $\{r \in \mathbb{Q} : r^2 < 2\} = \{r \in \mathbb{Q} : -\sqrt{2} < r < \sqrt{2}\}$, and thus is bounded above by $\sqrt{2}, 3, 4$ and by $-\sqrt{2}, -3, -4$ below.
(o) $x \in \mathbb{R} : x < 0\}$ is NBB, but bounded above by $0, 1, 2$.
(p) $\{1, \pi/3, \pi^2, 10\}$ is bounded above by $10, 11, 12$,and from below by $1, 0, -1$.
(q) $\{0, 1, 2, 4, 8, 16\}$ is bounded above by $16, 17, 18$ and below by $0, -1, -2$.
(r) $\bigcap_{n\in\mathbb{N}}(1 - 1/n, 1 + 1/n) = \{1\}$, and is bounded above $1, 2, 3$ and below by $1, 0, -2$.
(s) $\{1/n : n \in \mathbb{N}$ and n is prime$\}$, we are bounded from below by $1, 0, -1$ and in NBA.
(t) $\{x \in \mathbb{R} : x^3 < 8\} = \{x \in \mathbb{R} : x < 2\}$, we are bounded above by $2, 3, 4$ and NBB.
(u) $\{x^2 : x \in R\}$. This set is not bounded from above, but is bounded below by $0, -1, -2$.
(v) $\{\cos(n\pi/3) : n \in \mathbb{N}\} = \{1/2, -1/2, -1, 1\}$ and is bounded above by $1, 2, 3$ and below $-1, -2, -3$.
(w) $\{\sin(n\pi/3) : n \in \mathbb{N}\} = \{\sqrt{3}/2, 0, -\sqrt{3}/2\}$ is bounded above by $\sqrt{3}/2, 1, 2$ and below by $-\sqrt{3}/2, -1, -2$.

□

Question 2. *Give the supremum for items above (only go through (d)).*

Solution:
(a) $\sup([0, 1]) = 1$
(b) $\sup((0, 1)) = 1$
(c) $\sup(\{2, 7\}) = 7$
(d) $\sup(\{\pi, e\}) = \pi$

□