M413
Introduction to Analysis I
Assignment IX

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Problem 1. *Prove that* $\lim(-1)^n/n = 0$.

Discussion 1. We wish to show that $\forall \epsilon > 0$, $\exists N \ni\colon\ n > N \Rightarrow$

$$\left|\frac{(-1)^n}{n} - 0\right| = \frac{(-1)^n}{n} < \epsilon \Rightarrow \frac{1}{n} < \epsilon \Rightarrow n > \frac{1}{\epsilon}.$$

So, $N = 1/\epsilon$.

Proof. Let $\epsilon > 0$ and $N = 1/\epsilon$. Assume that $n > N$, that is $n > 1/\epsilon$. Then $|(-1)^n/n - 0| = |(-1)^n/n| = 1/n < \epsilon$.
$\therefore \lim(-1)^n/n = 0$. ☐

Problem 2. *Prove that* $\lim(2n-1)/(3n+2) = 2/3$

Discussion 2. We wish to show that $\forall \epsilon > 0$, $\exists N \ni\colon\ n > N \Rightarrow$

$$\left|\frac{2n-1}{3n+2} - \frac{2}{3}\right| < \epsilon$$

So we note that

$$\left|\frac{2n-1}{3n+2} - \frac{2}{3}\right| \rightarrow \left|\frac{6n-3-6n-4}{3(3n+2)}\right| \rightarrow \left|\frac{-7}{3(3n+2)}\right|,$$

and assuming that $n > -2/3$, we see that

$$\left|\frac{-7}{3(3n+2)}\right| = \frac{7}{3(3n+2} < \epsilon \Rightarrow \frac{7}{3} < \epsilon(3n+2) \rightarrow n > \frac{1}{3}\left(\frac{7}{3\epsilon} - 2\right).$$

So, N will be $\max\left\{\frac{-2}{3}, \frac{1}{3}\left(\frac{7}{3\epsilon} - 2\right)\right\}$.

Proof. Let $\epsilon > 0$ and let $N = \max\left\{\frac{-2}{3}, \frac{1}{3}\left(\frac{7}{3\epsilon} - 2\right)\right\}$. Assume that $n > N$. Then $|(2n-1)/(3n+2) - 2/3| = |-7/\left(3(3n+2)\right)| = 7/\left(3(3n+2)\right) < \epsilon$
$\therefore \lim(2n-1)/(3n+2) = 2/3$. ☐