M413
Introduction to Analysis I
Assignment I

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Theorem 1. *(DeMorgan's Law)*
Let X be a set and $\mathscr{A}$ be a family of subsets of X. Then

$$X \backslash \bigcap_{A \in \mathscr{A}} A = \bigcup_{A \in \mathscr{A}} (X \backslash A)$$

Proof. Let $x \in X$. Then $x \in X \backslash \bigcap_A A \Leftrightarrow x \notin \bigcap_A A \Leftrightarrow \exists A \in \mathscr{A}$ s.t. $x \in A \Leftrightarrow x \in \bigcup_A A \Leftrightarrow x \notin \bigcup_A (X \backslash A)$
$\therefore X \backslash \bigcap_{A \in \mathscr{A}} A = \bigcup_{A \in \mathscr{A}} (X \backslash A)$ □

Problem 2. *Prove that* $3 + 11 + ... + (8n - 5) = 4n^2 - n$.

Discussion 3. As with all induction proofs, we will start by showing the base case of $n = 1$. We will then assume true $\forall n \leq m$ where m is a fixed but unspecified natural number. At this point, we will then prove true for $m + 1$. It is important to note that $RHS_{m+1} = 4m^2 - 7m + 3$.

Proof. Suppose $n = 1$. Then $LHS = 3 = RHS = 4(1)^2 - 1$. So the proposition holds for $n = 1$. Now suppose the proposition holds $\forall n \leq m$. We will now show true for $m + 1$.
$LHS_{m+1} = 3 + 11 + ... + (8m - 5) + (8(m + 1) - 5) = 4m^2 - m + 8m + 3 = 4m^2 + 7m + 3$. As shown in discussion, this is RHS_{m+1}, therefore we conclude that the proposition holds $\forall n \in \mathbb{N}$ by mathematical induction. □

Problem 4. *Prove that* $7^n - 6n - 1$ *is divisible by* $8 \forall n \in \mathbb{N}$.

Discussion 5. We will start by showing the proposition holds true for $n = 1$. We will then assume the proposition holds $\forall n \leq m$ where m is a fixed but unspecified natural number. The notation $a|b$ as a divides b.

Proof. Suppose $n = 1$. Then $7^1 - 6(1) - 1 = 0$. However, $0 \not| 8$, so the proposition can not hold for all n. □