

# Abstract Algebra

## Assignment 5

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### Chapter 5 Revisited

# 17. Let

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix} \quad \text{and} \quad \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$$

compute

- (a)  $\alpha^{-1}$
- (b)  $\beta\alpha$
- (c)  $\alpha\beta$

*Solution:*

(a)  $\alpha^{-1} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}.$

(b) Note that in cycle notation,  $\alpha = (12)$  and  $\beta = (165432)$ , so  $\beta\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 2 & 3 & 4 & 5 \end{bmatrix}$

(c)  $\alpha\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 2 & 1 & 3 & 4 & 5 \end{bmatrix}.$

□

#18. Let

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{bmatrix} \quad \text{and} \quad \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$$

Write  $\alpha\beta$  and  $\beta\alpha$  as disjoint cycles, and products of disjoint cycles.

*Solution:*

$\alpha = (12345)(678)$ , and  $\beta = (23847)(56)$ .

$$\begin{aligned} \alpha\beta &= (12345)(678)(23847)(56) \\ &= \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 4 & 6 & 8 & 7 & 1 & 3 & 5 \end{bmatrix} \\ &= (12485736) \end{aligned}$$

$$\begin{aligned} \beta\alpha &= (23847)(56)(12345)(678) \\ \beta\alpha &= \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 7 & 6 & 1 & 2 & 4 & 5 \end{bmatrix} \\ \beta\alpha &= (13746285) \end{aligned}$$

□

# 19. Show that if  $H \leq S_n$ , then either  $\forall h \in H$ ,  $h$  is an even permutation or exactly half the members of  $H$  are even.

*Proof.* Assume that  $H \leq S_n$ , then  $H = S_{m \leq n}$ . Then  $\exists A_m$ , and by theorem,  $|A_m| = m!/2 \Rightarrow$  half the elements of  $H$  are even permutations.

□