Abstract Algebra
Assignment 3

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September 15, 2008

Theorem 1. *Let G be a group. Then, $|G| = 3 \Rightarrow G$ is cyclic.*

Proof. Let $|G| = 3$, and let $a \in G$ and $a \neq e$. Then $a^1 = a$ is unique. $a = a^2 \Rightarrow a = e$, so a^2 is unique. Similarly, $a^2 = a^3 \Rightarrow a = e$. So a^3 unique. Since $|G| = 3$, $a^4 = a$ and is therefore not unique. Therefore, G is cyclic. □

Problem 2. *Let $|x| = 40$. List all elements of order 10.*

Solution: $|x| = 40 \Rightarrow x^{40} = e$. So we need $|x^{10i}| = 40$, that is we need $10i|40$ in order to have $|x^i| = 10$. By through the powers of i from 1 to 40, I note that $x^i = 10$ only for $i \in \{4, 12, 16, 10, 28, 36\}$

□

Theorem 3. *Let a, b belong to a group. If $|a|$ and $|b|$ are relatively prime, then $\langle a \rangle \cap \langle b \rangle = \{e\}$.*

Proof. To simplify the notation let $A = \langle a \rangle$ and $B = \langle b \rangle$. Now note that $|a| = |A|$ and $|b| = |B|$. $\forall x \in A$, $|x| \mid |A|$. Let $x \in A \cap B$. Then. $|x| \mid |A|$ and $|x| \mid |B|$. Since $|A|$ and $|B|$ are relatively prime, they share no common factors, thus the only way for $|x|$ to divide $|A|$ and $|B|$ is for $|x| = 1 \Rightarrow x = e$. That is $A \cap B = \{e\}$. □