# 7.3 Estimating a Population Mean: $\sigma$ Known

In this section, we will again be working with confidence intervals and sample size determination, but here our objective is to estimate a population mean, $\mu$.

**ASSUMPTIONS:**

1. The sample is a simple random sample (All samples of the same size have an equal chance of being selected.)
2. The value of the population standard deviation $\sigma$ is known.
3. Either or both of these conditions is satisfied:
   i) the population is normally distributed, or
   ii) n > 30 (The sample has more than 30 values)

**The best point estimate of the population mean $\mu$ is $\overline{x}$.**

> **Example:** In list $L_1$ we have listed body temperatures from 106 people. The mean $\overline{x}$ of the sample is 98.20°. This is the best point estimate of the mean $\mu$ of all body temperatures.

Again, as in the previous section, to fine-tune our estimate, we may use a:
**CONFIDENCE INTERVAL:** a range (or interval) of values that is *likely* to contain the true value of the population mean.

**MARGIN OF ERROR (E) (or maximum error of the estimate)**: maximum likely difference between the observed sample mean $\overline{x}$ and the true value of the population mean $\mu$. It is calculated by multiplying the critical value and the standard deviation of the sample means:

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

**CONFIDENCE INTERVAL:** $\overline{x} - E < \mu < \overline{x} + E$

**CONFIDENCE INTERVAL LIMITS:** $\overline{x} - E$, and $\overline{x} + E$

**PROCEDURE FOR CONSTRUCTING A CONFIDENCE INTERVAL FOR $\mu$ (with known $\sigma$):**

1. Verify that the required assumptions are satisfied. (We have a simple random sample,
   $\sigma$ is known, and either the population appears to be normally distributed or $n > 30$.)
2. Find the critical value $z_{\alpha/2}$.
3. Evaluate the margin of error E.
4. Then using E and the sample mean $\overline{x}$, the confidence interval is:

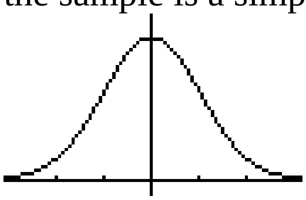
$$\overline{x} - E < \mu < \overline{x} + E$$

**ROUND-OFF RULE FOR CONFIDENCE INTERVALS:**

a) If data is original: one more decimal place than original values

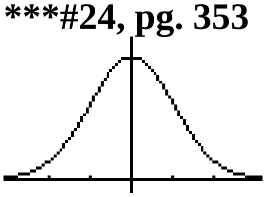
b) Summary values for data: same accuracy as $\overline{x}$

EXAMPLE: For the 106 body temperatures in list $L_1$, where $\overline{x} = 98.2$, construct the 98% confidence interval estimate of the mean body temperature for all healthy adults. Assume that the sample is a simple random sample and $\sigma$ is 0.6.



INTERPRET THE RESULTS:

*****#24, pg. 353**



INTERPRETATION of confidence interval: If you took many samples of size 50, 99% of the time the true mean of the population would be in that interval.

## USING THE TI- 83 TO CONSTRUCT A CONFIDENCE INTERVAL

First, either: Enter the data in a list, or
Have the summary statistics available.

Then, press STAT and select TESTS. Choose ZInterval (#7)
Then choose DATA (if you have raw data) or STATS (if you have summary statistics)
In either case, the TI-83 will prompt you for the necessary information

*****#24, pg. 353** Use the TI-83

## FINDING THE POINT ESTIMATE AND E FROM A CONFIDENCE INTERVAL

Given the confidence interval $\bar{x} - E < \mu < \bar{x} + E,$

The point estimate of $\mu$ is:

$$\bar{x} = \frac{\text{upper confidence interval limit} + \text{lower confidence interval limit}}{2}$$

The margin of error is:

$$E = \frac{\text{upper confidence interval limit} - \text{lower confidence interval limit}}{2}$$

*****#3, pg. 351 (find $\bar{x}$ and** E)

## Determining Sample Size Required to Estimate $\mu$

Solve $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ for $n$ to get formula for sample size: $n = [\frac{z_{\alpha/2} \cdot \sigma}{E}]^2$

where:

$z_{\alpha/2}$ = critical z-score based on the desired degree of confidence

E = desired margin of error

$\sigma$ = population standard deviation

****Always Round UP if the formula does not result in a whole number****

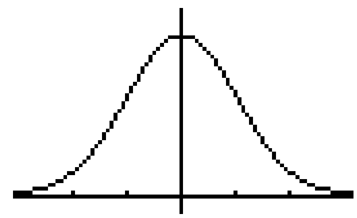
## **What if $\sigma$ is not known?**

1) Use the $range\, rule\, of\, thumb$: $\sigma \approx$ range/4

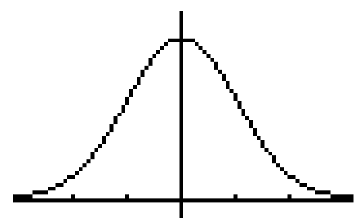
2) We often use $s$ from a small pilot test (n $>$ 30)

3) Estimate the value of $\sigma$ by using the results of some other study that was done earlier.

***#21, pg. 353



***#32, pg. 354



***#35, pg. 354