Electric potential and Faraday's law.

These notes are not a substitute for reading the text or doing problems in the text.

Electric potential.

Electric potential is related to work done in moving a charge in an electric field. Starting with the definition of work $W_{AB} = -\int_A^B \vec{F} \cdot d\vec{s}$ and using $\vec{F} = q\vec{E}$ we have $W_{AB} = -\int_A^B q\vec{E} \cdot d\vec{s}$ or $\Delta V = \frac{W_{AB}}{q} = -\int_A^B \vec{E} \cdot d\vec{s}$ where *ΔV* is the electrical potential difference between points A and B and is measure in volts.

- This means a volt is a Joule per coulomb; you should think of electric potential as energy needed to move a charge around.
- The potential is positive if we have to do work on a positive charge to move from A to B, negative if we get work out by moving from A to B. So moving a positive charge in a direction opposite the electric field which require work, lowering it with the electric field will make energy available.
- Notice *V* is a scalar because the dot product has gotten rid of the vector *F*, you do not have to break it into components. You *do* have to keep track of the sign of *q*.
- Note that if A and B happen to be the same point in a circuit this equation gives a rule called Kirchoff's loop rule: The sum of potentials in any current loop is zero if you return to the same point. This will be true only if there is no magnetic flux (see Faraday's law).

There are two special cases:

1. A uniform field (*E* is constant): $\Delta V = Ed\cos\theta$ where d is the distance between points A and B and *θ* is the angle between the path taken and E.
2. When you integrate the field of a point charge you get the electric potential for a point charge: $V = \frac{kq}{r}$. This is called the **absolute potential** because point B was taken to be infinity (*1/∞ = 0*).

The inverse operation of integration is the derivative so the definition of electric potential can also be written: $-\frac{\partial V}{\partial x} = E_x; -\frac{\partial V}{\partial y} = E_y; -\frac{\partial V}{\partial z} = E_z$

Faraday's Law

Magnetic flux has three components; area, angle (from the dot product) and magnetic field: $\Phi_B = \int \vec{B} \cdot d\vec{A}$. Faraday's law says that if any of these three components changes there will be a potential difference. In this case the potential, in volts, is called an emf or electromotive force although it is *not* a force. The emf is proportional to the rate of change of the magnetic flux: $\xi = -N\dfrac{d\Phi_B}{dt}$ where N is the number of loops of wire where the emf will appear.

- Something must *change* to create the voltage, either *B*, *A* or θ (the angle between *A* and *B*). Static fields and angles don't produce a voltage.
- Faraday's law creates a voltage. This voltage might produce a current but it won't if there is no circuit for the current to flow in.
- If the angle between *A* and *B* changes at a constant rate we have the generator equation: $\xi = -NBA\omega\cos(\omega t)$ where ω is the angular frequency of rotation.
- The negative sign in Faraday's law is an indication of Lenz's law which says that it requires input energy to create current flow. This means that the induced current which results from a change in the magnetic flux which *opposes* the motion which is creating this current. (You don't get something for nothing!) So it is harder to turn the crank of a generator what is generating current than one which is not connected to a circuit.
- A special case of Faraday's law is the moving bar: $\xi = Blv\cos\theta$ where *B* is the magnetic field, *l* is the length of the bar, *v* is the velocity and θ is the angle between *B* and *v*
- Examples of applications of Faraday's law include: the pickup on an electric guitar, swiping a credit card through a reader, the pickup of a cassette tape, the pickup of information on magnetic media such as hard drives, the embedded wire in the road which triggers a traffic signal when a car drives over it (thus changing the magnetic field), metal detectors at airports.

Maxwell's equations

Putting the definition of potential together with Faraday's law Maxwell found: $\oint \vec{E}\cdot d\vec{s} = -N\dfrac{d\Phi_B}{dt}$. Maxwell also corrected Ampere's law to include the possibility that a changing electric flux would create a magnetic field: $\oint \vec{B}\cdot d\vec{s} = \varepsilon_o\mu_o\dfrac{d\Phi_E}{dt} + \mu_o I$. These two equations together with two Gauss's laws $\oint \vec{E}\cdot d\vec{A} = \dfrac{q}{\varepsilon_o}$ and $\oint \vec{B}\cdot d\vec{A} = 0$ are called Maxwell's equations. All of the properties of electricity and magnetism are contained in these four equations.