Student understanding of the ideal gas law, Part I: A macroscopic perspective

Christian H. Kautz,[a] Paula R. L. Heron, Michael E. Loverude,[b] and Lillian C. McDermott
Department of Physics, University of Washington, Seattle, Washington 98195

(Received 6 April 2005; accepted 29 July 2005)

Our findings from a long-term investigation indicate that many students cannot properly interpret or apply the ideal gas law after instruction in introductory physics and chemistry as well as more advanced courses. The emphasis in this paper is on the concepts of pressure, volume, and temperature at the macroscopic level. We describe some serious conceptual and reasoning difficulties that we have identified. Results from our research were applied in the design of a curriculum that has helped improve student understanding of the ideal gas law.

[DOI: 10.1119/1.2049286]

I. INTRODUCTION

The Physics Education Group at the University of Washington (UW) has been engaged in a long-term project to improve student learning in thermal physics. This paper is the first of two in which we report results from our investigation of student ability to interpret and apply the ideal gas law, $PV=nRT$.[1] The emphasis is on the macroscopic variables of pressure, volume, and temperature, and their relation to one another through the ideal gas law. The interpretation of n (the number of moles) and related microscopic processes is discussed in Ref. 2. Insights gained from this research have guided us in the design of instructional strategies to improve student learning.

The understanding of thermal phenomena by pre-university students has been probed extensively.[3] Of the relatively few studies at the university level, the two most directly relevant to our research were conducted by Rozier and Viennot in France and Meltzer in the United States.[4,5]

Our investigation involved more than 1000 students at four universities. Most were enrolled in lecture and laboratory sections of algebra-based physics or in a sophomore-level thermal physics course at UW. Other participants were students in calculus-based physics at the University of Cincinnati, the University of Maryland, and Syracuse University. Students in introductory chemistry at Syracuse also participated. All of these courses discussed the ideal gas law, the first law of thermodynamics, and the relation between the internal energy and the temperature of an ideal gas.

Our decision to focus on the ideal gas law was motivated by preliminary results from an earlier study on student understanding of the first law of thermodynamics.[6] We found that many students invoked the ideal gas law inappropriately when trying to respond to questions that required application of the first law. Not only did students reveal an inadequate understanding of the first law, especially of the role of work, but they also demonstrated a profound misunderstanding of the equation of state for an ideal gas. These findings prompted us to undertake a concurrent investigation related to the ideal gas law.[7]

II. METHODS OF INVESTIGATION

The present study began with individual interviews conducted with 45 UW students who were enrolled in or who had completed the second quarter of the algebra-based course ($N=16$) or the sophomore-level thermal physics course ($N=29$).[8] Many had taken a year of chemistry. Almost all performed at or above average in their respective classes. Most interviews occurred in the weeks preceding and following the students' final examinations. The adiabatic and isobaric processes that formed the basis for the questions were inspired by the work of Rozier and Viennot.[4]

In the interviews, the students were shown a plastic bicycle pump filled with air. They were asked to predict what would happen to the temperature of the air if the opening of the pump were sealed and the piston were pushed in rapidly. In this approximately adiabatic process, the temperature would rise, a prediction that most of the students made but could not justify. Almost none mentioned the first law of thermodynamics and the concept of work. They relied only on the ideal gas law, which is inadequate for predicting the rise in temperature. Students also often made incorrect microscopic arguments. In some interviews, students were given a second task in order to probe their understanding of pressure. They were asked whether or not it would be possible to decrease the volume of air in the pump without changing the pressure and to explain their reasoning. Many of the students maintained that any decrease in volume would result in an increase in pressure, not recognizing that a decrease in volume could be accomplished by a decrease in temperature.

The nature and prevalence of the errors made by the students motivated us to probe more deeply into their understanding of the macroscopic variables of pressure, temperature, and volume. We designed several types of written problems for this purpose. All involved qualitative questions for which explanations of reasoning were required. Our analysis of the results guided the design of curriculum to address specific difficulties.

Our group assesses student learning through pretests and post-tests. Usually, pretests are given after a standard treatment of a topic but before research-based instruction. When possible, the post-tests are administered on examinations or on ungraded quizzes.[9] The post-tests require application of the concepts in situations somewhat different from those encountered on the pretests or during instruction. Memorization is of insufficient help. The results can be compared with those from corresponding pretests. (Similarity between a pretest and post-test has almost no effect on student perfor-

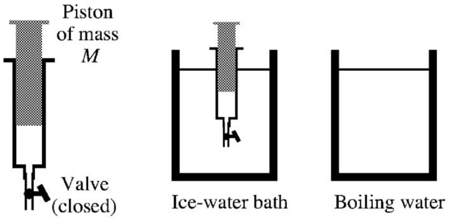

Fig. 1. The vertical-syringe problem. A syringe that contains an ideal gas and has a frictionless piston of mass M is moved from an ice-water bath to a beaker of boiling water, where it comes to thermal equilibrium. Students are asked if the final pressure and volume of the gas are greater than, less than, or equal to the initial pressure and volume, respectively. They are asked to explain their reasoning.

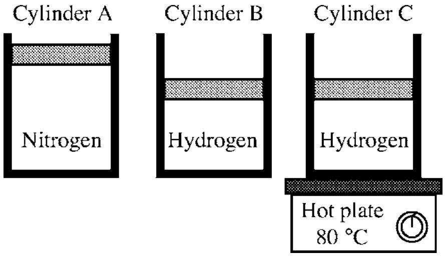

Fig. 2. The three-cylinders problem. Three identical cylinders are filled with unknown quantities of ideal gases. The cylinders are closed with identical frictionless pistons of mass M. Cylinders A and B are in thermal equilibrium with the room at 20 °C, and cylinder C is kept at a temperature of 80 °C. The students are asked whether the pressure of the nitrogen gas in cylinder A is greater than, less than, or equal to the pressure of the hydrogen gas in cylinder B, and whether the pressure of the hydrogen gas in cylinder B is greater than, less than, or equal to the pressure of the hydrogen gas in cylinder C. They are asked to explain their reasoning.

mance.) Alternatively, comparisons can be made with posttests from classes that have had only standard instruction.

Our experience with other topics has shown that results of our assessments are similar for similar classes with similar instruction.[10] Therefore, we have combined results from multiple sections of the same course, rounded the numbers of students, and given the percentages of correct and incorrect responses to the nearest 5%. As a guide for the development and assessment of curriculum, this range of reproducibility in student performance has proven to be sufficient.

III. PROBLEMS DESIGNED TO PROBE STUDENT UNDERSTANDING

Below we discuss three examples of problems designed to probe student understanding of the ideal gas law from a macroscopic perspective. For each problem, we give a proper response and note the prevalence of correct answers. A more detailed analysis follows in Sec. IV.

A. Vertical-syringe problem

Students consider a glass syringe that contains a gas that they are told to treat as ideal (see Fig. 1). The syringe is sealed with a frictionless piston of mass M and is initially in thermal equilibrium with an ice-water bath. The syringe is then placed in a beaker of boiling water, where it reaches thermal equilibrium. The students are asked to compare the pressures and volumes of the gas in the initial and final equilibrium states.[11] To answer correctly, they must recognize that the force exerted by the atmosphere and the gravitational force on the piston are the same in the initial and final states. Therefore, for the piston to be in mechanical equilibrium both before and after the expansion, the force exerted by the enclosed gas, and hence the pressure, must be the same. From the ideal gas law, the increase in temperature must be associated with an increase in volume. The results are shown in Table I.

In the introductory physics courses ($N>1000$), the responses did not depend strongly on the type of course (algebra- or calculus-based), timing (before or after instruction), or setting (lecture or laboratory). About 30% of the students realized that the initial and final pressures are equal. (About 10% gave correct reasoning.) In introductory chemistry after instruction on the ideal gas law, only about 15% said that the pressures were equal. In the thermal physics course, about 45% gave the correct answer. In all courses, the most common incorrect answer was that the final pressure would be greater than the initial pressure.

B. Three-cylinders problem

In Fig. 2 are three identical cylinders (A, B, and C) that contain unspecified quantities of different ideal gases. They are sealed with identical frictionless pistons. Cylinders A and B are at the same temperature, but hold different gases. Their pistons are at different heights. Cylinders B and C contain the same type of gas but are at different temperatures. Their pistons are at the same height. The students are asked to compare the pressures of the gases in cylinders A and B, and in B and C. A correct answer requires noting that the atmospheric pressure and the weight of the piston are the same for each cylinder. Therefore, all must have the same pressure. The results are summarized in Table II.

For the case in which the pistons are at different heights (cylinders A and B), about 30% of the students in introductory physics ($N\sim250$) stated correctly that the pressures are the same. Of the remaining students, about the same percent-

Table I. Results from the vertical-syringe problem (Fig. 1).

	Introductory physics $N>1000$	Introductory chemistry $N\sim95$	Thermal physics $N\sim35$
$P_{\text{final}}=P_{\text{initial}}$ (correct)	30%	15%	45%
$P_{\text{final}}>P_{\text{initial}}$	60%	70%	40%
Other or no answer	10%	15%	20%

Table II. Results from the three-cylinders problem (Fig. 2).

	Introductory physics $N \sim 250$	Thermal physics $N \sim 65$
Pistons at different heights		
$P_A = P_B$ (correct)	30%	45%
$P_A > P_B$	35%	30%
$P_A < P_B$	30%	15%
undecided or no answer	5%	10%
Gases at different temperatures		
$P_B = P_C$ (correct)	40%	70%
$P_B > P_C$	0%	0%
$P_B < P_C$	55%	30%
undecided or no answer	5%	~0%
All three cylinders		
$P_A = P_B = P_C$ (correct)	15%	40%

age said that either A or B had a higher pressure. For gases at different temperatures (cylinders B and C), about 40% recognized that the pressures are equal. Almost all of the rest said that the pressure in C (higher temperature) is greater. Correct comparisons of the pressures in all three cylinders were made only by about 15% of the introductory students. About 40% of the thermal physics students ($N \sim 65$) correctly ranked all three pressures. The errors were similar to those in the introductory courses.

C. Insulated-cylinder problem

Several small masses are gradually added to the top of a frictionless piston that seals an insulated cylinder that contains an ideal gas (see Fig. 3). The students are asked how (if at all) the pressure, temperature, and volume change. They need to recognize that positive work is done on the gas as the piston moves down. (Students were not expected to justify that the piston would drop.) Because the cylinder is insulated, little or no heat transfer occurs. By the first law of thermodynamics, the internal energy of the gas inside the cylinder must increase. The internal energy is proportional to the temperature, which will therefore also increase. In the final position, the piston will be lower than initially, indicating a decreased volume of the gas. Because the system of piston and masses has a greater weight than before and is again at rest, the pressure of the gas must have increased.

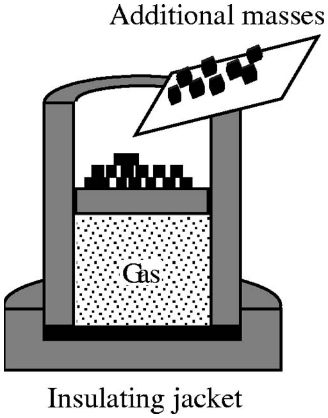

Fig. 3. The insulated-cylinder problem. A cylinder with a frictionless piston contains an ideal gas. The cylinder is placed in an insulating jacket and small masses are added. The students are asked whether the pressure, temperature, and volume of the gas will increase, decrease, or remain the same. They are asked to explain their reasoning.

This problem was administered to about 180 students in the algebra-based course. About 70% of the students realized that the volume would decrease. Only about 10% gave a correct answer for the temperature.

IV. ANALYSIS OF INCORRECT RESPONSES

The many incorrect student responses indicate that after standard instruction many students in introductory physics and chemistry, as well as more specialized courses in thermal physics, have serious difficulties in interpreting and applying the ideal gas law. In the following analysis, these difficulties are organized into categories that are not mutually exclusive. Conceptual, reasoning, and mathematical difficulties often are intertwined and cannot be completely separated.

A. Incorrect responses related to interdependence of the variables

In none of the tasks discussed in Sec. III is the ideal gas law sufficient to predict whether, or by how much, any of the variables would change. In making predictions, many students focused on a single relation between two quantities that is valid only if the others are constant.

Incorrectly assuming that P is always $\propto 1/V$. The following quote is from an interview with a student in the thermal physics course who attempted to apply the ideal gas law to an adiabatic compression. "[The temperature] … is going to remain the same, because the pressure goes up as the volume decreases, and air is approximately an ideal gas in these conditions, so the temperature should not change in this case… An ideal gas follows the equation $PV = nRT$." Similarly, some students assumed an inverse proportionality between pressure and volume in cylinders A and B in the three-cylinders problem (which would be valid for equal amounts of gas) and concluded that the pressure is lower in cylinder A, which has the higher piston.

Incorrectly assuming that P is always $\propto T$. In the vertical-syringe problem, many students argued that higher temperature implies greater pressure. For example, one student wrote, "P increases because $P = nRT/V$. So as T increases, P increases." This student and many others ignored the fact that the volume must be the same in both the initial and final states for P and T to be directly proportional. (Almost all of these students then correctly answered the subsequent question by stating that the volume would increase.) In comparing cylinders B and C on the three-cylinders problem, many students treated the pressure and temperature as proportional. Because cylinder C was at a higher temperature, they expected its pressure to be greater. These results confirm the findings of Rozier and Viennot that students often do not properly interpret multivariable equations.[4] Moreover, it was clear from our analysis of their explanations that many were not relating the mathematical formalism to the real world. However, difficulties with the ideal gas law extend far beyond formalism and control of variables in experiments.

B. Incorrect responses related to pressure

The errors made by the many students who incorrectly compared pressures fall into two broad categories. One reflects lack of understanding of mechanical equilibrium; the other seems to result from flawed microscopic models.

Incorrectly relating gas pressure to mechanical equilibrium. Many students seemed unable to relate the mechanical equilibrium of a piston to the force exerted on it by the enclosed gas and, hence, the gas pressure. Even being asked to draw a free-body diagram for the piston in the vertical syringe in the final equilibrium state did not help them. Those who considered mechanical conditions often focused on the height, rather than the weight, of the piston. (Their free-body diagrams often showed a nonzero net force.) The following response by a student in the thermal physics course was typical: "The piston is raised higher. Greater force against piston." Similar explanations were given by students who claimed that the pressure is higher in cylinder A than in B in the three-cylinders problem. A tendency to associate the force exerted on an object with its vertical position occurs in many other situations.[10]

Incorrectly relating gas pressure to a microscopic model. Between 15% and 25% of the introductory students supported the incorrect prediction that the pressure in the vertical syringe would increase when it was placed in boiling water by referring explicitly to the motion of gas particles. A typical statement was "The molecules are traveling faster, hitting the sides of [the syringe] more, thus creating higher pressure."[12] Such references often revealed incorrect or incomplete microscopic models for pressure.[2]

C. Incorrect responses related to temperature

The errors made by students who incorrectly compared initial and final temperatures on the insulated-cylinder problem fall into two broad categories. One reflects confusion among heat, temperature, and internal energy; the other results from flawed microscopic models.

Incorrectly assuming that temperature does not change in an adiabatic process. On the insulated-cylinder problem, about 35% of the students argued that the insulation keeps the temperature constant with no reference to heat transfer. For example, one student wrote "In a perfectly insulated system, the temperature should remain constant." According to these students, thermal insulation implies constant temperature even if other changes are made to the system. Some students specified that the insulating jacket keeps the temperature constant by not allowing any heat transfer to occur.[13] For example, one student said "Temperature remains the same. The insulating jacket (if it works perfectly) keeps heat from going in or out."

Some students seemed to interpret a thermal insulator as a device that actively counteracts any changes in temperature in a system (rather than just preventing heat transfer).[14] For example, a student who seemed to recognize that the compression tends to increase the internal energy of the gas wrote, "[The temperature will] remain the same; the insulation will absorb the heat."

Difficulties in distinguishing between heat and temperature have been well documented, especially among young students, usually in the context of calorimetric experiments with liquids and solids.[15] Confusion between these concepts also exists at the university level.[16,17] Reference to *heat* as a state (rather than a process) quantity has been noted.[18] In this study, confusion among heat, temperature, and internal energy appeared to underlie many student claims that temperature does not change in an adiabatic process.

Incorrectly interpreting temperature at the microscopic level. Incorrect microscopic models led students to wrong predictions. Many attributed increases in temperature and internal energy to collisions among the molecules.[2,6] Others assumed that temperature increases with the density of particles. Conceptual difficulties with temperature at the microscopic level are discussed in greater detail in Ref. 2.

D. Incorrect responses related to volume

Difficulties with the concept of volume, especially in differentiating it from the amount of a substance, have been extensively studied among young students.[19] We observed similar difficulties at the university level in courses for elementary school teachers, in courses for students underprepared in science, and in the algebra-based course.[20]

Not distinguishing between volume and amount of gas. On the vertical-syringe problem, the inability of some students to apply the correct concept suggested a failure to distinguish amount (mass) from space (volume). About 20% of the introductory students and about 15% of the thermal physics students predicted that the volume would be the same when the syringe was moved from ice water to boiling water. On the insulated-cylinder problem, about 20% of the introductory students said the volume would be the same after the masses were added. Students who predicted that the volume would not change often explained that no air had entered or left the syringe. The following statement is an example: "The volume is equal to V_{initial} because it's still the same amount in the syringe." Others claimed that the volume would stay the same even if the position of the piston changed. For some students, the terms volume and number of particles seemed synonymous: "The volume of the gas is equal to V_{initial}. It doesn't matter whether the piston moved or not. There is no gas leaving or entering, so although the actual shape may change, the number of gas particles remains constant."

Initially, we had assumed that only academically weak students had trouble with the concept of volume at such a basic level. However, the average course grade of students with this difficulty was not significantly different from the class average. Apparently, inability to distinguish between volume and amount of gas is not confined to less able students.

V. RESEARCH-BASED INSTRUCTION TO IMPROVE STUDENT UNDERSTANDING

Certain difficulties do not disappear during standard instruction. If sufficiently serious, they may preclude development of a functional understanding of the material. One approach for helping students construct a coherent conceptual framework is through instruction that explicitly addresses such difficulties. Instructional strategies that have been expressly designed for this purpose can be quite effective.[21] An iterative process, in which ongoing assessment plays a critical role, has been successfully applied in our development of the *Heat and Temperature* module in *Physics by Inquiry*.[22] This laboratory-based curriculum helps students separate the concepts of heat transfer and temperature in solids and liquids. Distinguishing among heat transfer, temperature, and thermodynamic work in gases is considerably more complex.[5,6] Because these concepts are strongly linked, the difficulties they present cannot be treated separately but must be addressed together.

Drawing on results from research, we have developed tutorials on the ideal gas law and the first law of thermodynamics.[23] As with other topics, each tutorial sequence consists of a pretest, worksheet, homework, and post-test.[24] The pretests help students recognize what they are expected to understand and provide a baseline for the instructor. During the 50-min tutorial sessions, which optimally have 20–25 students, groups of 3–4 students collaborate on worksheets that guide them by carefully structured questions through the reasoning required for a functional understanding. Teaching assistants ask additional questions to help students arrive at answers for themselves. Tutorial homework helps reinforce student learning from the worksheets. Assessment is by post-tests.

We have also developed a tutorial laboratory experiment on the ideal gas law. Here the term *tutorial* indicates that the experiment embodies research-based instructional strategies similar to the tutorials. As students make quantitative measurements of the variables in the ideal gas law, they are asked to respond to qualitative questions on the interpretation and application of the concepts at a macroscopic level. The tutorial and tutorial laboratory experiment were designed to be alternative forms of instruction and are independent. Some students in this investigation participated in both.

A. Tutorial on the ideal gas law

The tutorial on the ideal gas law is intended to help students deepen their understanding of the relationship of gas pressure to mechanical equilibrium, as well as to the other variables in the ideal gas law.[25] We have found that tasks that elicit student difficulties often can be used to address these difficulties. We therefore chose as the context for the tutorial, an isobaric process like the one in the vertical-syringe problem. The tutorial helps students draw on their knowledge of mechanics to develop a conceptual understanding of gas pressure. The treatment is based on the definition, $P=F/A$, and is entirely macroscopic.

The tutorial is divided into four parts. In the first, students are guided to recognize that a movable piston can keep the pressure of a gas in a vertical cylinder at a constant value.[26] In the second part, they are led to recognize that an increase in temperature does not necessarily imply an increase in pressure. The third part of the tutorial introduces PV diagrams as a useful representation of the states and processes of an ideal gas, in which the pressure is related to the volume explicitly. The last part, which is described in Ref. 2, helps students to recognize the substance independence of the ideal gas law.[2] The first three parts are summarized below.

Relating gas pressure to mechanical equilibrium. The students consider a gas-filled cylinder that is closed at the top by a massive but frictionless piston. They are asked to draw a free-body diagram for the piston, which should include an identification of the objects on which and by which each force is exerted. The students then apply Newton's second law and the definition of pressure to find an expression that relates the pressure of the gas to the given quantities (that is, the mass and cross-sectional area of the piston, and the atmospheric pressure).

A subsequent problem addresses the prevalent belief that different positions of the piston correspond to different pressures. After deriving an algebraic expression for the pressure in the cylinder, the students consider a second cylinder with a different sample of an ideal gas. The two cylinders and their pistons are identical, but the piston in the second is shown in the diagram at a greater height than that in the first. The students are asked whether the pressure in the second cylinder is greater than, less than, or equal to that in the first and to explain their reasoning. Through discussions among themselves and with the instructors they determine that the pressure is the same in both.

Relating gas pressure to temperature and volume. The students consider a situation similar to the vertical-syringe problem. The main objective is to recognize that an increase in temperature does not necessarily imply an increase in pressure, which depends on the external conditions. The students are asked whether the temperature, pressure, and volume of gas in a cylinder have increased, decreased, or remained the same after it has been removed from an ice-water bath and placed in boiling water. They check their answers for consistency with the definition of pressure and with the ideal gas law and reiterate which variables were kept constant and which changed. They now recognize that a change in temperature at constant pressure is possible and that this change requires a change in the volume. The result is a displacement of the piston. (Most students realize that the volume changes, but initially expect the pressure to change as well.)

The tutorial contains a fictional dialogue in which a student states that an increase in temperature must lead to an increase in pressure. A second student claims that the volume does not change because no gas enters the system and also concludes that the pressure increases. In deciding whether or not they agree, the students identify the inconsistencies in the arguments.

Relating PV diagrams to the ideal gas law. To help students improve their understanding of the ideal gas law, the tutorial provides practice in drawing and interpreting PV diagrams for different processes. The students come to recognize that pressure, volume, and temperature are interdependent, and that implicit information about the temperature can be obtained by applying the ideal gas law.

B. Tutorial laboratory experiment on the ideal gas law

The experiment was designed to help students develop operational definitions for volume and pressure and relate these variables to the devices used to measure them. The

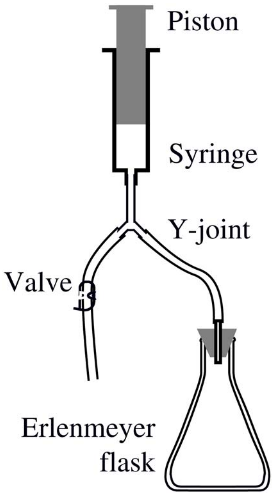

Fig. 4. Schematic diagram of laboratory apparatus.

experiment has several parts. Some involve the collection and analysis of quantitative data, while others are qualitative and allow students to make observations that conflict with their expectations. Most of the data are taken with a closed system that consists of an air-filled syringe and a small Erlenmeyer flask to which a pressure gauge can be connected (see Fig. 4). Paper-and-pencil exercises emphasize conceptual understanding.

First the students are asked to decide which of the variables (P, V, n, or T) is described by the graduations on the syringe. They learn to distinguish between the volume and the amount of gas (n) by noting that one quantity changes when the piston is moved while the other does not. They next identify the forces on the piston (with the syringe in the orientation shown in Fig. 4) and express the relationship among these forces using Newton's second law. We emphasize the importance of mechanical equilibrium and the role of atmospheric pressure by having the students repeat this exercise with the syringe turned "tip-up." (Many expect the piston to drop out of the syringe when its orientation is changed.) After calculating the pressure of the enclosed air with and without additional weights on the piston, the students compare their results to the readings on the pressure gauge and critique a statement by a fictional student that the pressure inside is equal to atmospheric pressure.

The students next perform three experiments, the removal of a small amount of air from the system at constant pressure, an (approximately) isobaric expansion (through immersion of the flask in hot water), and an isochoric increase in temperature. Students are referred to the discussion of the forces on the piston if they are surprised that the reading on the pressure gauge does not change during the isobaric process. In the isochoric process, the students find that they must exert an increasing force on the piston to maintain its position. They then collect quantitative data by repeating the experiment with a rigid container attached to a pressure gauge. They estimate a value for absolute zero and show that the Kelvin unit of temperature must be used for calculations involving the ideal gas law.

C. Tutorial on the first law of thermodynamics

The tutorial on the first law of thermodynamics consists of four parts.[27] In the first, the students apply the definition of mechanical work to a block moving up and down an incline. They then consider a gas enclosed in a cylinder sealed with a frictionless piston and are asked about the direction of the force exerted by the piston on the gas. They decide how the piston would have to move in order to perform positive or negative work on the gas.

In the second part, students are reminded that the change in internal energy of a gas in a thermally insulated system is equal to the work done on the gas. They consider a gas in an insulated cylinder in which the piston is moved inward. They are asked to identify flaws in the reasoning of a fictional student who claims that "the volume of the gas decreases, but the pressure increases. Therefore, by the ideal gas law, the temperature remains the same."

In the third part of the tutorial, the students consider the same sample of gas at room temperature. The piston is now locked in place and the insulating jacket removed. The cylinder is put into boiling water and the gas is allowed to reach thermal equilibrium. The students are asked about changes in the temperature, internal energy, volume, and pressure of the gas. After sketching the PV diagram for the process, they determine that no work was done on the gas. The term *heat transfer* is introduced for the energy transfer that occurred.

Changes in internal energy due to work and heat transfer are combined in the fourth part of the tutorial. The students consider an uninsulated cylinder with a piston free to move that is placed in an ice-water bath and allowed to come to equilibrium. The piston is then slowly pressed inward in such a way that the gas is always in equilibrium with the ice-water bath. In this isothermal process, both work and heat transfer must be taken into account. To help students confront and resolve any confusion between heat and temperature, they are asked if they agree or disagree with a fictional student who claims that "the temperature doesn't change, it is an isothermal process. Therefore, the heat transfer must be zero." The students are expected to recognize that the second statement is incorrect.

D. Use of tutorials in interactive tutorial lectures

We have not implemented the tutorials on thermal physics in the type of small-group environment that has proved effective for other topics. Instead, these tutorials have formed the basis for interactive lectures, during which neighboring students work collaboratively.[28] At intervals ranging from 5–20 min, the instructor initiates a discussion with the class. No extra class time was involved.

VI. PROBLEMS DESIGNED TO ASSESS STUDENT LEARNING

We administered several post-tests on course examinations after students had worked through the tutorials and tutorial laboratory experiment. To be able to make pretest/post-test comparisons, we gave the same problems to students with only standard instruction. Performance on other tasks that we had used to probe student understanding provided an additional benchmark for assessing learning. Although the data presented in Sec. III are from several universities, the results of the problems described in this section are only from the University of Washington (UW), where we could more readily ensure consistency in the analysis of examination questions and document the conditions of their implementation.

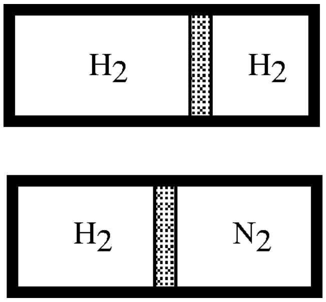

Fig. 5. The double-chamber problem. A cylinder is divided into two chambers by a freely sliding piston of mass M. In each of the situations, both chambers contain unknown amounts of ideal gases and the piston is at rest. Both gases are at the same temperature. The students are asked to compare the pressures of the gases in the two chambers and to explain their reasoning.

A. Double-chamber problem

The double-chamber problem was given as a post-test early in the development of the tutorial on the ideal gas law and tutorial laboratory experiment. The problem involves a long horizontal cylinder that is divided into two chambers by a freely sliding piston (see Fig. 5). Students are asked to compare the pressures of the gases in the two chambers, which differ in some respect. In one situation, the piston is not at the center. Thus there are different volumes for the two gas samples. In another situation, the two chambers contain equal volumes of different gases. In both situations, the pressures in both chambers are equal because the cylinder is horizontal. As shown in Table III, students who participated in the interactive tutorial lecture did somewhat better than those who had standard instruction. The best overall performance was from students who participated in both the interactive tutorial lecture and the tutorial laboratory experiment.

B. Two-cylinders problem

The two cylinders shown in Fig. 6 contain an equal number of moles of the same ideal gas. The pressures are equal but the temperatures are not. The position and mass of the movable piston in cylinder 1 are shown. The students are asked which of the three movable pistons (A, B, or C) is used to close cylinder 2. Compared with the piston in cylinder 1, A has a smaller mass, B has an equal mass, and C has a greater mass. Therefore, the correct choice is B. The students also are asked to describe the height of this piston relative to that in cylinder 1. Because both cylinders contain the same number of moles, the volume of the sample in cylinder 2 (which is at a higher temperature) is greater. Therefore, piston B should be higher. Students who took the two-cylinders problem as a post-test had worked through later versions of the ideal-gas-law tutorial than students who took the double-chamber problem. As shown in Table IV, students who participated in the interactive tutorial lectures or worked through the tutorial laboratory experiment performed better than those who had standard lecture instruction only.[29]

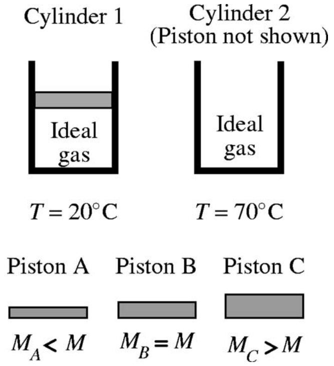

Fig. 6. The two-cylinders problem. Two identical cylinders with frictionless pistons contain an equal number of moles of the same ideal gas at the same pressure. Cylinder 1, which has a piston of mass M, is at room temperature. Cylinder 2 is at a temperature of 70 °C. The piston that closes cylinder 2 is not shown. In the bottom diagram are three movable pistons of different masses that fit cylinder 2. Students are asked to choose the correct piston (A, B, or C) for cylinder 2 and to explain their reasoning.

C. Insulated-cylinder problem

The insulated-cylinder problem in Fig. 3 has served as a post-test for the effectiveness of the first-law tutorial in addressing difficulties with temperature related to thermal insulation. (It was not administered in classes in which it had been a pretest.) After interactive tutorial lectures, half of the students made correct predictions with correct reasoning ($N \sim 195$) and another 25% with incorrect or incomplete reasoning. In contrast, after standard lecture instruction only

Table III. Results from the double-chamber problem (Fig. 5) after different types of instruction at UW. The percentage gives the fraction of students who correctly compared pressures with correct reasoning for each situation.

	Introductory algebra-based course			
	After standard lecture instruction		After interactive tutorial lectures and no standard lecture instruction	
	No lab $N\sim 55$	Tutorial lab experiment $N\sim 190$	No lab $N\sim 70$	Tutorial lab experiment $N\sim 60$
Different volumes	30%	60%	40%	60%
Different gases	30%	50%	45%	65%

Table IV. Results from the two-cylinders problem (Fig. 6) after different types of instruction at UW.

	Introductory algebra-based course		
	After standard lecture instruction		After interactive tutorial lectures and no standard lecture instruction
	No lab $N \sim 35$	Tutorial lab experiment $N \sim 20$	No lab $N \sim 125$
Piston of same mass (correct)			
with correct reasoning	30%	50%	70%
without correct reasoning	15%	15%	10%
Piston of greater mass	30%	20%	10%
Piston of less mass	20%	20%	10%

about 10% responded correctly and none gave correct reasoning ($N \sim 180$). The percentage who claimed that the temperature of the gas would not change because of the insulation decreased to about 15%, an improvement from 35% after standard instruction.

D. Horizontal-syringe problem

We used a problem based on a sealed, insulated, horizontal syringe containing an ideal gas to assess student understanding of the concept of volume. The piston is pulled outward and held in the new position. Students are asked whether the final volume of the gas is greater than, less than, or equal to its initial value. In a section of a calculus-based course that had not had tutorials, about 10% of the students gave incorrect answers ($N \sim 100$). On the vertical-syringe problem, about 20% in the algebra-based course predicted that the volume would remain the same, often saying that the system is closed or that no gas could enter or leave. Although these percentages are small, lack of a functional understanding of volume cannot be ignored. The tutorial and tutorial laboratory experiment seem to have helped. When the horizontal-syringe problem was asked on a post-test in the algebra-based course, all but two students ($N \sim 120$) said that the volume would increase.

VII. CONCLUSION

The results from this investigation indicate that after instruction in introductory physics and chemistry, as well as in more advanced courses, many students cannot properly interpret the macroscopic variables of pressure, temperature, and volume in an ideal gas. We found that difficulties with mechanics severely limited the ability of students to relate the ideal gas law to physical situations. They frequently confused the concepts of heat, temperature, and internal energy. Underlying many of their difficulties with the macroscopic variables was often a profound misunderstanding of microscopic models and processes.[2]

There is evidence that the interactive tutorial lectures and tutorial laboratory experiment, both separately and in combination, helped students improve their understanding of the macroscopic concepts of pressure and volume in a gas. Use of the tutorials in the lectures has not significantly reduced the overall pace of instruction. However, the topics addressed are sufficiently important that some decrease in the breadth of coverage and illustrations of problem solving is warranted. Research-based instruction has helped students recognize the role of work in changing the temperature of a gas—a critical step toward separating the concepts of heat transfer, temperature, and internal energy. The ability to distinguish these concepts is critical for predicting the behavior of ideal gases, in which changes in temperature can occur adiabatically and heat transfers can take place without changes in temperature. Learning to make such distinctions also is very important for understanding a much wider range of thermal phenomena than those that involve ideal gases.

ACKNOWLEDGMENTS

This investigation has been a collaborative effort by many members of the Physics Education Group. Substantive contributions were made by Peter S. Shaffer and Matthew Cochran. Mark N. McDermott collaborated in the development of the tutorial experiment. The authors deeply appreciate the cooperation of the faculty whose classes participated in the study at UW, the University of Cincinnati, the University of Maryland, and Syracuse University. The authors also gratefully acknowledge support from the National Science Foundation through the Division of Undergraduate Education and the Physics Division.

---

[a] Present address: Hamburg University of Technology, 21071 Hamburg, Germany.

[b] Present address: Department of Physics, California State University, Fullerton, CA 92834.

[1] In most of the physics courses in this study, the ideal gas law is expressed as $PV=nRT$, where n is the number of moles. In a few cases, the law was expressed as $PV=NkT$, where N is the number of gas particles.

[2] C. H. Kautz, P. R. L. Heron, P. S. Shaffer, and L. C. McDermott, “Student understanding of the ideal gas law, Part II: A microscopic perspective,” Am. J. Phys. **73**, 1064–1071 (2005).

[3] For a brief annotated bibliography that includes some studies among young students, as well as a few at the university level, see L. C. McDermott and E. F. Redish, “Resource letter: PER-1: Physics education research,” Am. J. Phys. **67**, 755–767 (1999).

[4] S. Rozier and L. Viennot, “Students’ reasonings in thermodynamics,” Int. J. Sci. Educ. **13**, 159–170 (1991).

[5] D. E. Meltzer, “Investigation of students’ reasoning regarding heat, work, and the first law of thermodynamics in an introductory calculus-based general physics course,” Am. J. Phys. **72**, 1432–1446 (2004).

[6] M. E. Loverude, C. H. Kautz, and P. R. L. Heron, “Student understanding of the first law of thermodynamics: Relating work to the adiabatic compression of an ideal gas,” Am. J. Phys. **70**, 137–148 (2002).

[7] C. H. Kautz, “Identifying and addressing student difficulties with the ideal gas law,” Ph.D. dissertation, Department of Physics, University of Washington, 1999 (unpublished).

[8] For a succinct description of this research method, see L. C. McDermott, “Millikan Lecture 1990: What we teach and what is learned—Closing the gap,” Am. J. Phys. **59**, 301–315 (1991). This article includes references to papers that give specific examples.

[9] The quizzes are taken seriously by most students, as is discussed in L. G. Ortiz, P. R. L. Heron, and P. S. Shaffer, “Investigating student understanding of static equilibrium: Predicting and accounting for balancing,” Am. J. Phys. **73**, 545–553 (2005).

[10] See, for example, M. E. Loverude, C. H. Kautz, and P. R. L. Heron, “Helping students develop an understanding of Archimedes’ Principle, Part I: Research on student understanding,” Am. J. Phys. **71**, 1178–1187 (2003).

[11] The students were asked to compare the initial and final equilibrium states to avoid the issue of whether the process itself could be considered quasistatic.

[12] Note the similarity between this statement and the one made by a student in Ref. 5 in the context of an isothermal process. Meltzer, however, interprets this statement primarily as evidence for student difficulties with the microscopic model of temperature (rather than pressure).

[13] Meltzer has observed a similar type of incorrect student reasoning in the context of an isothermal process, that is, that the absence of a temperature change implies that there is no heat transfer (Ref. 5). See also Ref. 6.

[14] For these students, the insulation seems to assume the role of a thermal reservoir that supplies or absorbs heat in order to keep the system’s temperature constant.

[15] See, for example, G. Erickson and A. Tiberghien, “Heat and temperature,” in *Children’s Ideas in Science*, edited by R. Driver, E. Guesne, and A. Tiberghien (Open University Press, Milton Keynes, 1985), and the chapter on “Heating” in R. Driver, A. Squires, P. Rushworth, and V. Wood-Robinson, *Making Sense of Secondary Science* (Routledge, London, 1994).

[16] M. L. Rosenquist, “Improving preparation for college physics of minority students aspiring to science-related careers: Investigation of student difficulties and development of appropriate curriculum,” Ph.D. dissertation, Department of Physics, University of Washington, 1982 (unpublished).

[17] M. J. Cochran, “An investigation of student understanding of the second law of thermodynamics and the underlying concepts of heat, temperature and thermal equilibrium,” Ph.D. dissertation, Department of Physics, University of Washington, 2005 (unpublished).

[18] P. H. van Roon, H. F. van Sprand, and A. H. Verdonk, “‘Work’ and ‘heat’: On a road towards thermodynamics,” Int. J. Sci. Educ. **16**, 131–144 (1994). See also Refs. 2 and 4–6.

[19] See, for example, K. C. de Berg, “Student understanding of the volume, mass, and pressure of air within a sealed syringe in different states of compression,” J. Res. Sci. Teach. **32**, 871–884 (1995).

[20] L. C. McDermott, L. K. Piternick, and M. L. Rosenquist, “Helping minority students succeed in science, Part I. Development of a curriculum in physics and biology,” J. Coll. Sci. Teach. **9**, 136–140 (1980), and Refs. 10 and 16.

[21] For examples, see the articles by L. C. McDermott and others in the Physics Education Group that have been published in Am. J. Phys., several of which are included in Ref. 3.

[22] L. C. McDermott and the Physics Education Group at the University of Washington, *Physics by Inquiry* (Wiley, New York, 1996). See the *Heat and Temperature* module.

[23] L. C. McDermott, P. S. Shaffer, and the Physics Education Group at the University of Washington, *Tutorials in Introductory Physics* (Prentice Hall, Upper Saddle River, NJ, 2002).

[24] For a more complete description of the tutorials and their implementation, see L. C. McDermott, Oersted Medal Lecture 2001: “Physics education research—The key to student learning,” Am. J. Phys. **69**, 1127–1137 (2001).

[25] See Ref. 23, pp. 227–230.

[26] The conditions necessary for a movable piston to ensure constant pressure include the absence of friction and negligible acceleration of the piston at all times. In the tutorial, students assume that there is no friction between the piston and the container walls. The conditions for quasistatic changes are beyond the scope of the tutorial. All situations considered are therefore equilibrium states before and after any changes take place.

[27] See Ref. 23, pp. 231–235.

[28] For a more detailed description of an interactive tutorial lecture, see P. R. L. Heron, M. E. Loverude, P. S. Shaffer, and L. C. McDermott, “Helping students develop an understanding of Archimedes’ Principle, Part II: Development of research-based instructional materials,” Am. J. Phys. **71**, 1188–1195 (2003).

[29] The results for students who had participated in the interactive tutorial lectures and who had done the tutorial laboratory experiment were identical to those for students who had participated in the interactive tutorial lectures only.