T101 SECTION 3-5 MENTAL MATHEMATICS AND ESTIMATION FOR WHOLE NUMBER OPERATIONS

What is mental mathematics?

What is Computational Estimation?

Proficiency in mental mathematics can help you in your everyday estimation skills. You need to be able to judge the reasonableness of your answers.

I. MENTAL MATHEMATICS – ADDITION

A. Adding from the left

$$\begin{array}{r} 69 \\ +\ 53 \\ \hline \end{array}$$

Sixty plus fifty is 110.
Nine plus three is 12.
So 110 plus 12 is 122.

B. Breaking up and Bridging

$$\begin{array}{r} 42 \\ +\ 53 \\ \hline \end{array}$$

Forty-two plus fifty is 92.
92 plus 3 more is 95.

C. Trading Off

$$\begin{array}{r} 67 \\ +\ 23 \\ \hline \end{array}$$

Sixty-seven is only 3 away from seventy.
So take 3 from the 23 (leaving only 20)
and give it to the 67 to make 70. Now
70 plus 20 more is 90.

D. Using Compatible Numbers (Sums that are easy to calculate)

$$\begin{array}{r} 60 \\ 20 \\ 140 \\ 80 \\ +\ 30 \\ \hline \end{array}$$

Try to make 100's or 10's or other
"nice" numbers. 140 plus the 60 is 200.
20 plus the 80 is 100, so now I am up
to 300. 300 plus 30 more is 330.

E. Making Compatible Numbers

$$\begin{array}{r} 25 \\ +\ 79 \\ \hline \end{array}$$

25 adds "nicely" to 75 (instead of
79) to make 100. But I am still
4 short, so 100 plus 4 is 104.

II. MENTAL MATHEMATICS – SUBTRACTION

A. Breaking up and Bridging

$$\begin{array}{r} 57 \\ -\ 33 \\ \hline \end{array}$$

Start with 57 and take away 30 to get 27. Then 27 minus 3 more is 24.

B. Trading Off

$$\begin{array}{r} 67 \\ -\ 48 \\ \hline \end{array}$$

It is easier to subtract when your minuend is a "nice" number (a multiple of 10). So I will increase 48 by 2 to 50 and to maintain the difference, I will also add 2 to 67 to get 69. Now, 69 minus 50 is 19.

C. Drop the Zeros

$$\begin{array}{r} 8\,700 \\ -\ \ 500 \\ \hline \end{array}$$

Since both numbers have 2 zeros at the end, I can drop the 2 zeros. Then I just subtract 87 minus 5 to get 82 and then put the 2 zeros back on to get 8200.

D. Adding Up or "Cashier's Algorithm"

You owe $11 for your groceries. You use a $50 bill. How much should you receive in change? The cashier starts at your bill and counts up to how much money you gave the cashier – thus giving you the difference or your change.

The cashier counts	$11	$12	$13	$14	$15	$20	$30	$50
Money you receive	$0	$1	$1	$1	$1	$5	$10	$20 = _____

So, $11 (payment) + $39 (change) = $50 (bill you paid with)

III. MENTAL MATHEMATICS – MULTIPLICATION

A. Front End Multiplication

$$\begin{array}{r} 69 \\ \times \quad 3 \\ \hline \end{array}$$

Sixty times three is 180.
Nine times three is 27.
180 plus 20 is 200 plus 7 more is 207.

B. Using Compatible Numbers

2 x 9 x 5 x 20 x 5

Try to make 100's or 10's or other "nice" numbers. Twenty times 5 is 100. Two times 5 is 10. 100 times 10 is 1000. Finally, 1000 times 9 is 9000.

C. Thinking Money (How many cents?)

$$\begin{array}{r} 68 \\ \times \quad 5 \text{ (nickels)} \\ \hline \end{array}$$

I can think of the 5 as a nickel. If I have 68 nickels, this is the same as 34 dimes which is 34 x 10 = 340 cents

$$\begin{array}{r} 32 \\ \times \quad 50 \text{ (half dollars)} \\ \hline \end{array}$$

32 half dollars = _____ dollars

_____ dollars = _____ cents

IV. MENTAL MATHEMATICS – DIVISION

A. Breaking up the Dividend

$7\overline{)5642}$

Break up the dividend so that it is the sum of numbers that the divisor can easily divide. I will break 5642 up into 5600 + 42. Now, 7 goes into 5600, 800 times and 7 goes into 42, 6 times. So my quotient is 800 plus 6 = 806.

B. Using Compatible Numbers

$4\overline{)136}$

→ $4\overline{)120 + 16}$

I want to try to make the dividend the sum of two numbers that are divisible by the divisor. 136 can break up into 120 plus 16. Now 4 divides 120, 30 times and divides 16, 4 times.
So my quotient is 30 plus 4 = 34

$8\overline{)232}$

→ $8\overline{)240 - 8}$

V. COMPUTATIONAL ESTIMATION – ADDITION

A. Front-End

```
  456
  641
+ 392
```

I will add from the front – the 100's column. 400 plus 600 plus 300 is 1300. To make my estimate better, I will look at the 10's columns. 50 plus 40 is about 100 and 90 is about 100, so 1300 plus 200 more is 1500.

B. Grouping to "Nice" Numbers

```
  23
  39
  32
  64
+ 49
```

I want to FORCE "nice" sums.
39 plus the 64 is about 100.
23 plus 32 is about 50.
49 is about 50.
That totals about 200.

C. Clustering

```
 5956
 6241
 6312
+ 5892
```

Each of these numbers is about 6000. 6000 times 4 is 24,000. This is about 24,000.

D. Rounding (Round each number to the highest place value of the smallest number.)

```
 7956
+ 5192
```

Round to 1000s. 8000 plus 5000 is 13,000

```
 1267
-  510
```

Round to 100s.

E. Using the Range (Get a Low Estimate and get a High Estimate)

Round both DOWN

Round both UP

	Low Estimate	High Estimate
641		
+ 372	+ _____	+ _____.

Thus the range for this answer is from _____ to _____.

VI. COMPUTATIONAL ESTIMATION – MULTIPLICATION AND DIVISION

A. Front-end Multiplication

$$\begin{array}{r} 769 \\ \times \quad 3 \\ \hline \end{array}$$

Starting from the front,
700 times 3 is 2100.
60 times 3 is 180 or about 200.
2100 plus 200 is about 2300.
I'm only looking for an estimate – so I'll stop there.

B. Using Compatible Numbers

$5 \overline{)5\,6\,4\,2}$

I want a number close to 5642 that 5 could easily go into.
Hmmm, how about 5500?
5500 divided by 5 is 1100.

TAKE HOME PROBLEMS

MENTAL MATHEMATICS – ADDITION

A. Adding from the left

$$\begin{array}{r} 31 \\ +\ 48 \\ \hline \end{array}$$

B. Breaking up and Bridging

$$\begin{array}{r} 31 \\ +\ 58 \\ \hline \end{array}$$

C. Trading Off

$$\begin{array}{r} 38 \\ +\ 48 \\ \hline \end{array}$$

D. Making Compatible Numbers

$$\begin{array}{r} 130 \\ +\ 78 \\ \hline \end{array}$$

MENTAL MATHEMATICS – SUBTRACTION

A. Breaking up and Bridging

$$\begin{array}{r} 98 \\ -\ 45 \\ \hline \end{array}$$

B. Trading Off

$$\begin{array}{r} 73 \\ -\ 27 \\ \hline \end{array}$$

C. Drop the Zeros

$$\begin{array}{r} 6\,200 \\ -\ 700 \\ \hline \end{array}$$

MENTAL MATHEMATICS - MULTIPLICATION

A. Front End Multiplication

$$\begin{array}{r} 31 \\ \times \quad 8 \\ \hline \end{array}$$

B. Thinking Money (How many cents?)

$$\begin{array}{r} 24 \\ \times \quad 25 \text{ (quarters)} \\ \hline \end{array}$$

24 quarters. = _____ half dollars

_____ half dollars = _____ dollars

_____ dollars = _____ cents

MENTAL MATHEMATICS - DIVISION

A. Breaking up the Dividend

$6\overline{)3\ 6\ 4\ 8}$

B. Using Compatible Numbers

$8\overline{)1\ 0\ 4}$ ⟶ $8\overline{)\qquad\qquad}$

COMPUTATIONAL ESTIMATION - ADDITION

A. Front-End

$$\begin{array}{r} 519 \\ 382 \\ +\ 652 \\ \hline \end{array}$$

B. Grouping to "Nice" Numbers

$$\begin{array}{r} 32 \\ 42 \\ 55 \\ 28 \\ +\ 42 \\ \hline \end{array}$$

COMPUTATIONAL ESTIMATION - MULTIPLICATION AND DIVISION

A. Front-end Multiplication

$$\begin{array}{r} 531 \\ \times \quad 8 \\ \hline \end{array}$$

(500 x 8) =

(30 x 8) = + _____ .

B. Using Compatible Numbers

$6\overline{)4\ 3\ 7\ 2}$ ⟶ $6\overline{)\qquad\qquad}$