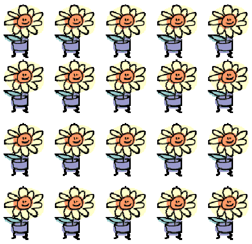
T101 SECTION 3-3 MULTIPLICATION AND DIVISION OF WHOLE NUMBERS

I. MULTIPLICATION OF WHOLE NUMBERS

Suppose a garden contains 4 rows of flowers with 5 flowers in each row. How many flowers are in the garden?



5 + 5 + 5 + 5 = ________

or “5, 4 times over”

or “5 times 4" = 5 x 4 = ________

The advantage of the multiplication notation is obvious when the number of addends becomes large. THINK? 55 rows of 5 flowers each? WOW!

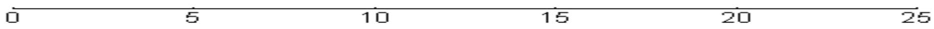
A. REPEATED ADDITION MODEL

7 x 5 = ______ + ______ + ______ + ______ + ______ = ___________

Or = ______ + ______ + ______ + ______ + ______ + ______ + ______ = ___________

B. NUMBER LINE

To model 5 x 4, make 4 jumps of size 5.

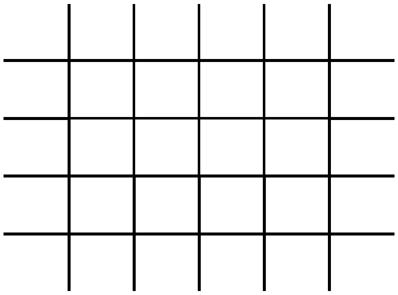


C. ARRAY MODEL

An array consists of objects in equal-sized rows. We can illustrate using sticks crossing to create intersection points thus forming an array. The number of intersection points is the product.

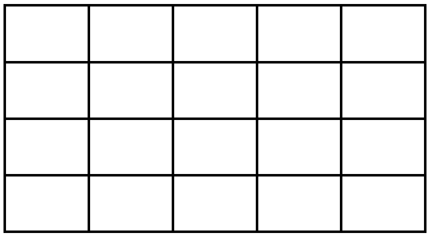
(2 x 6 = 12)

(5 x 4 = 20)



D. AREA MODEL

An Area Model is shown by a grid. The number of unit squares required to fill in the grid is the product. (5 x 4 = 20)



DEFINITION OF MULTIPLICATION OF WHOLE NUMBERS

For any whole number **a** and **n, where n ≠ 0**, we define "a times n" as:

$a \cdot n =$ that is, **n** terms of **a**

If **n** = 0 , then **a** $\cdot$ 0 = 0 (That is, 0 terms of **a**)

E. CARTESIAN-PRODUCT MODEL (A set of ordered pairs)

To illustrate this model we will make use of a TREE DIAGRAM.

Example: Suppose you have two pairs of pants (jeans and khakis) and 3 shirts (red, blue and green) in your closet, how many outfits can you make?

Each outfit can be written as an ordered pair – (jeans, blue shirt). The set of all ordered pairs forms the Cartesian Product.

Notation: If B is the number of pants and C is the number of shirts, then
B X C, "B cross C", is the number of outfits.

F. PROPERTIES OF MULTIPLICATION OF WHOLE NUMBERS

1. Closure Property of Multiplication of Whole Numbers

For any whole numbers a and b, ____

This simply means that given any set, the set is closed under multiplication if the product of any two or more elements is also a member of the set.

Which of the following are closed under multiplication?
{2, 4, 6, 8, …}   {1, 2, 3, 4}   {1, 3, 5, 7, …}   {0}   {0, 1}

2. Commutative Property of Multiplication of Whole Numbers

For any whole numbers a and b, ______________________________

3. Associative Property of Multiplication of Whole Numbers

For any whole numbers a, b, and c, ____

4. Identity Property of Multiplication of Whole Numbers

There is a unique whole number ______, called the *multiplicative identity* such that for any

whole number a, ____

5. Zero Property of Multiplication of Whole Numbers

For any whole number a, ____

6.

a. Distributive Property of Multiplication over Addition
For any whole numbers a, b, and c, ____

b. Distributive Property of Multiplication over Subtraction
For any whole numbers a, b, and c, ____

Name the demonstrated property – Use the complete name & spelling counts.

$4 \cdot 9 = 9 \cdot 4$ $(3 \cdot 8) \cdot 5 = 3 \cdot (8 \cdot 5)$ $1 \cdot 47 = 47 \cdot 1 = 47$

$0 \cdot k = k \cdot 0 = 0$ $(a \cdot b) \cdot c = c \cdot (a \cdot b)$ $(3 + 4) \cdot 8 = 3 \cdot 8 + 4 \cdot 8$

G. MORE ON THE DISTRIBUTIVE PROPERTY

Looking at the Distributive Property "in reverse" we get: $ab + ac = a(b + c)$
This is commonly called ***factoring.***

Factor each of the following:

$2x + 4$

$a^2 - ab$

$32 \cdot 9 + 32$

$12xy^2 - 9x^2y$

II. DIVISION OF WHOLE NUMBERS

A. SET (PARTITION) MODEL (Similar to Dealing Cards)

Suppose that I have 15 cookies I want to share between me and my two friends. How many cookies will each of us receive?

ME FRIEND 1 FRIEND 2

15 ÷ 3 = ______

B. MISSING-FACTOR MODEL

Another strategy for dividing 15 cookies among the three of us is to answer the division problem by using our knowledge of multiplication. $\boxed{} \bullet 3 = 15$, so $15 \div 3 = \boxed{}$

This again introduces the student to **fact families:**

$5 \cdot 3 = 15$
$3 \cdot 5 = 15$
$15 \div 3 = 5$
$15 \div 5 = 3$

> DEFINITION OF DIVISION OF WHOLE NUMBERS
> For any whole numbers **a** and **b**, with **b ≠ 0**, **a** ÷ **b** = **c**, if and only if, **c** is the unique whole number such that **b** · **c** = **a**.

In a division problem, a ÷ b = q, commonly written as: $\begin{array}{r} q \\ b\overline{)\,a} \end{array}$

a is called the ______________________________,
b is called the ______________________________, and
q is called the ______________________________.

C. REPEATED-SUBTRACTION MODEL

Suppose that we have 15 cookies that will be bagged in groups of three for our bake sale. How many baggies will we need?

15 - 3 = _____ - 3 = ______ - 3 = ______ - 3 = ______ - 3 = ______
(1 bag) (2) (3) (4) (5)

Then count how many bags of three cookies you were able to create.

Or 15 - 3 - 3 - 3 - 3 - 3 = 0 (15 - "five" 3's = 0) So, 15 ÷ 3 = ________

Before we talk more about division, let's do a simple problem just to review:

$5\overline{)\,32}$ Then how do we check our answer?

D. THE DIVISION ALGORITHM

We know that division of whole numbers is not always perfect (*remainders!*).

THE DIVISION ALGORITHM

Given any whole numbers **a** (dividend) and **b** (divisor), with **b ≠ 0**, there exists unique whole numbers **q** (quotient) and **r** (remainder) such that

$$\begin{array}{r} q \quad r \\ b\overline{)\;a\quad} \end{array}$$

________________________ with $0 \le r < b$.

If 123 is divided by a number and the remainder is 13, what are the possible divisors?

E. DIVISION INVOLVING A ZERO (VERY IMPORTANT!!!)

6 ÷ 0 = ☐ (Think, what number times zero gives you 6? 0 · ☐ = 6)

n ÷ 0 = ________________________

0 ÷ 8 = ☐ (Think, what number times 8 gives you 0? 8 · ☐ = 0)

0 ÷ n = ________________________

0 ÷ 0 = ☐ (Think, what number times 0 gives you 0? 0 · ☐ = 0)

0 ÷ 0 = ________________________

or ________________________

F. DIVISION BY 1

n ÷ 1 = _________ (Think, what number times 1 gives you n? 1 · ☐ = n)