

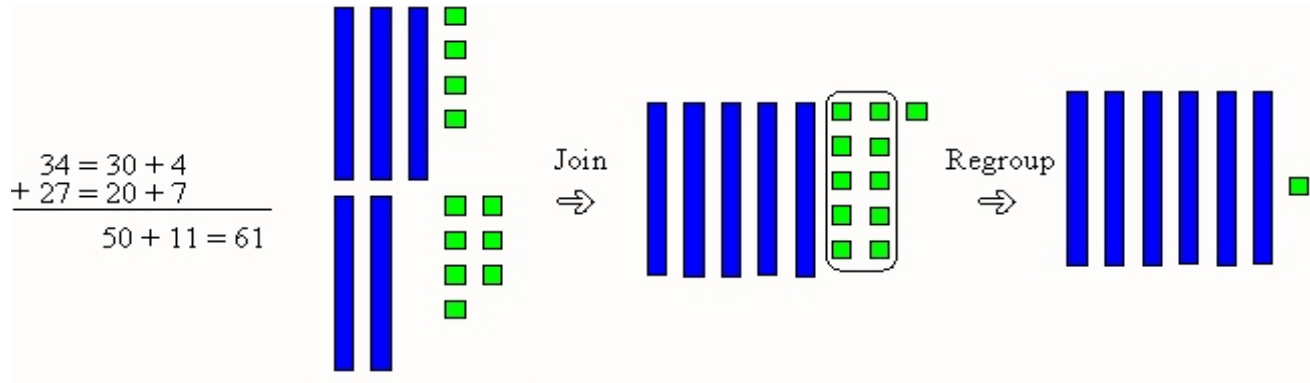
What is an algorithm?

Paper and pencil algorithms must be taught *developmentally*; that is, they must proceed from the concrete stage (using blocks, chips, abacus, and counters) to the abstract stage (following rote rules).

I. ADDITION ALGORITHMS

A. Establishing the Standard Algorithm

CONCRETE: Use Base-10 Blocks to add



$$\begin{array}{r} 34 = 30 + 4 \\ + 27 = 20 + 7 \\ \hline 50 + 11 = 61 \end{array}$$

ABSTRACT ALGORITHMS

Expanded Algorithm

Tens	Ones	
3	4	
+ 2	7	
		(Add the ones)
		(Add the tens)

Standard Algorithm

$$\begin{array}{r} 34 \\ + 27 \\ \hline \end{array}$$

B. LEFT-TO-RIGHT ALGORITHM FOR ADDITION

$$\begin{array}{r} 568 \\ +757 \\ \hline \end{array}$$

or

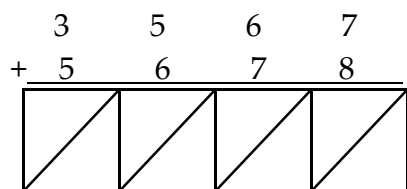
$$\begin{array}{r} 568 \\ +757 \\ \hline \end{array}$$

You try:

$$\begin{array}{r} 9076 \\ +4689 \\ \hline \end{array}$$

$$\begin{array}{r} 9076 \\ +4689 \\ \hline \end{array}$$

C. LATTICE ALGORITHM FOR ADDITION



D. SCRATCH ALGORITHM

$$\begin{array}{r} 35 \\ 79 \\ +26 \\ \hline \end{array}$$

$$\begin{array}{r} 378 \\ 653 \\ +594 \\ \hline \end{array}$$

II. SUBTRACTION ALGORITHMS

A. Establishing the Standard Algorithm

CONCRETE: Use Base-10 Blocks to subtract

The diagram shows four stages of the subtraction process using base-10 blocks:

- Stage 1:** A vertical number line shows 56 (5 tens and 6 ones). Below it, the text reads: "Represent 56. Can not take away 29 from this representation." The blocks consist of five blue tens rods and six green ones units.
- Stage 2:** One ten rod is broken into ten ones units. Below it, the text reads: "Trade one ten for ten ones." The blocks now consist of four blue tens rods, one green ten rod, and six green ones units.
- Stage 3:** Two tens rods and nine ones units are crossed out with diagonal lines. Below it, the text reads: "Take away 29." The remaining blocks are two blue tens rods, one green ten rod, and seven green ones units.
- Stage 4:** The final result is shown as two blue tens rods and seven green ones units. Below it, the text reads: "Represent answer as 27."

ABSTRACT ALGORITHMS

Expanded Algorithm

$$\begin{array}{r} 56 \\ - 29 \\ \hline \end{array} \Rightarrow \begin{array}{r} (50 + 6) \\ - (20 + 9) \\ \hline \end{array} \Rightarrow \begin{array}{r} (40 + 16) \\ - (20 + 9) \\ \hline (20 + 7) = 27 \end{array}$$

Standard Algorithm

$$\begin{array}{r} 56 \\ - 29 \\ \hline \end{array}$$

$$\begin{array}{r} 243 \\ - 61 \\ \hline \end{array}$$

B. EQUAL ADDENDS ALGORITHM

The equal addends algorithm for subtraction is based on the fact that the difference between the two numbers does not change *even if we add the same amount to both numbers.*

For instance:

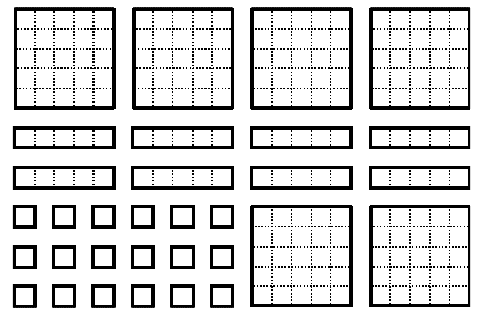
$$\begin{array}{r} 15 \\ - 9 \\ \hline \end{array} \quad \begin{array}{l} (+1) \longrightarrow \\ (+1) \longrightarrow \end{array} \quad \begin{array}{r} 16 \\ - 10 \\ \hline \end{array}$$

Even more elaborate:

$$\begin{array}{r} 213 \\ - 42 \\ \hline \end{array} \quad \begin{array}{l} (+ \quad) \\ (+ \quad) \end{array} \quad \begin{array}{r} (+ \quad) \\ (+ \quad) \end{array} =$$

III. ADDITION AND SUBTRACTION IN BASES OTHER THAN TEN

We can use the same techniques from above to help us in our addition and subtraction in other bases. We start with Base 5 and then elaborate to other bases.



BASE-5 BLOCKS

$$32_{\text{five}} + 14_{\text{five}}$$

$$41_{\text{five}} - 34_{\text{five}}$$

SCRATCH ADDITION

$$\begin{array}{r} 24_{\text{five}} \\ + 34_{\text{five}} \\ \hline \end{array}$$

$$\begin{array}{r} 746_{\text{eight}} \\ + 552_{\text{eight}} \\ \hline \end{array}$$

LATTICE ADDITION

$$\begin{array}{r} 1 \quad 4 \quad 3_{\text{five}} \\ + 2 \quad 3 \quad 4_{\text{five}} \\ \hline \end{array}$$

ABSTRACT ALGORITHM (DO NOT CONVERT TO BASE 10 FIRST!!!):

$$\begin{array}{r} 43_{\text{five}} \\ + 32_{\text{five}} \\ \hline \end{array}$$

$$\begin{array}{r} 32_{\text{five}} \\ - 13_{\text{five}} \\ \hline \end{array}$$

$$\begin{array}{r} 500_{\text{six}} \\ - 53_{\text{six}} \\ \hline \end{array}$$

TAKE HOME PROBLEMS:

ADD USING THE TWO DIFFERENT LEFT-TO-RIGHT ALGORITHMS

$$\begin{array}{r} 9076 \\ +4689 \\ \hline \end{array}$$

$$\begin{array}{r} 9076 \\ +4689 \\ \hline \end{array}$$

ADD USING LATTICE ALGORITHM

$$\begin{array}{r} 5 \quad 6 \quad 5 \quad 9 \\ + \underline{7 \quad 3 \quad 4 \quad 8} \end{array}$$

ADD USING SCRATCH ALGORITHM

$$\begin{array}{r} 87 \\ 65 \\ + \underline{49} \end{array}$$

ADD USING SCRATCH ADDITION

$$\begin{array}{r} 143 \text{ five} \\ + \underline{234} \text{ five} \end{array}$$

ADD OR SUBTRACT IN THE GIVEN BASE

$$\begin{array}{r} 32 \text{ five} \\ + \underline{13} \text{ five} \end{array}$$

$$\begin{array}{r} 111 \text{ five} \\ - \underline{13} \text{ five} \end{array}$$