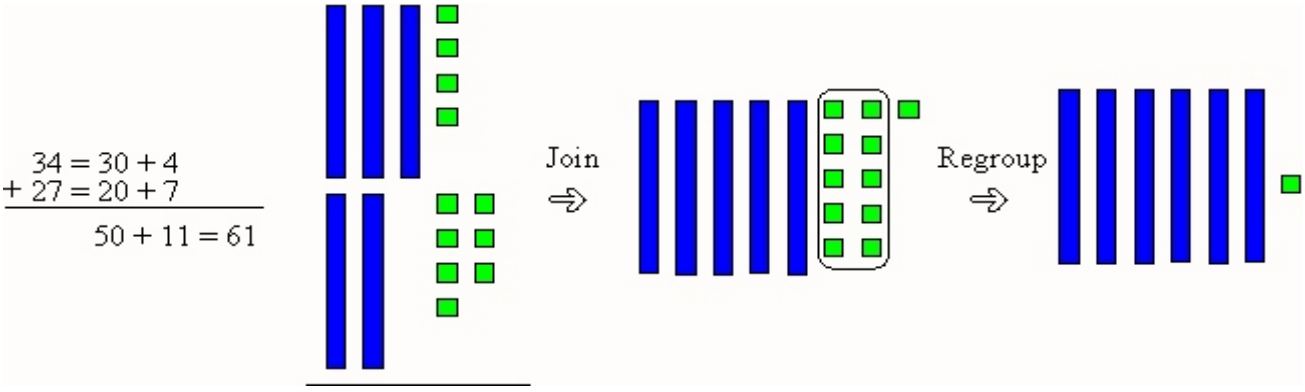
What is an algorithm?

Paper and pencil algorithms must be taught *developmentally*; that is, they must proceed from the concrete stage (using blocks, chips, abacus, and counters) to the abstract stage (following rote rules).

I. ADDITION ALGORITHMS

A. Establishing the Standard Algorithm

CONCRETE: Use Base-10 Blocks to add



ABSTRACT ALGORITHMS

Expanded Algorithm

Tens	Ones	
3	4	
+ 2	7	
		(Add the ones)
		(Add the tens)

Standard Algorithm

$$\begin{array}{r} 34 \\ +27 \\ \hline \end{array}$$

B. LEFT-TO-RIGHT ALGORITHM FOR ADDITION

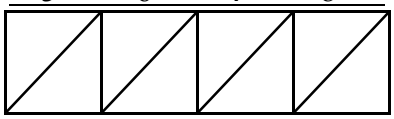
```
  568          or          5 6 8
+757                     +7 5 7
```

You try:

```
  9076                   9 0 7 6
+4689                  +4 6 8 9
```

C. LATTICE ALGORITHM FOR ADDITION

```
   3   5   6   7
+  5   6   7   8
```



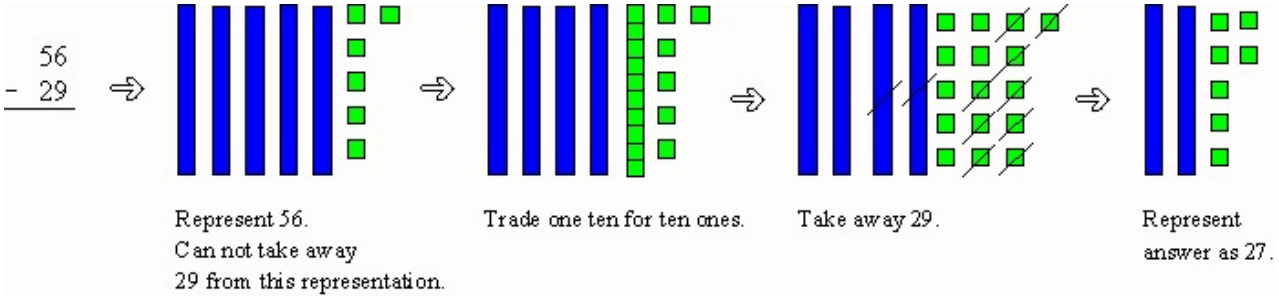
D. SCRATCH ALGORITHM

```
  35          378
  79          653
+ 26        + 594
```

II. SUBTRACTION ALGORITHMS

A. Establishing the Standard Algorithm

CONCRETE: Use Base-10 Blocks to subtract



ABSTRACT ALGORITHMS

Expanded Algorithm

$$\begin{array}{r} 56 \\ -\ 29 \end{array} \Rightarrow \begin{array}{r} (50 + 6) \\ -\ (20 + 9) \end{array} \Rightarrow \begin{array}{r} (40 + 16) \\ -\ (20 + \ \ 9) \\ \hline (20 + \ \ 7) = 27 \end{array}$$

Standard Algorithm

$$\begin{array}{r} 56 \\ -\ 29 \end{array} \qquad \begin{array}{r} 243 \\ -\ \ 61 \end{array}$$

B. EQUAL ADDENDS ALGORITHM

The equal addends algorithm for subtraction is based on the fact that the difference between the two numbers does not change *even if we add the same amount to both numbers.*

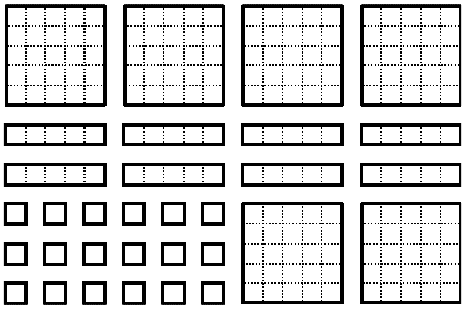
For instance:

$$\begin{array}{rcl} 15 & (+1) \longrightarrow & 16 \\ \underline{-\ 9} & (+1) \longrightarrow & \underline{-\ 10} \end{array}$$

Even more elaborate:

$$\begin{array}{rcl} 213 & (+\quad) & (+\quad) \\ \underline{-\ 42} & (+\quad) - \underline{\qquad\qquad} & (+\quad) - \end{array}$$

III. ADDITION AND SUBTRACTION IN BASES OTHER THAN TEN



We can use the same techniques from above to help us in our addition and subtraction in other bases. We start with Base 5 and then elaborate to other bases.

BASE-5 BLOCKS

$32_{five} + 14_{five}$ $41_{five} - 34_{five}$

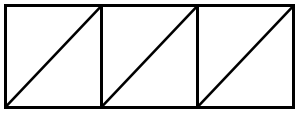
SCRATCH ADDITION

$$\begin{array}{r} 24_{five} \\ +\ 34_{five} \\ \hline \end{array}$$

$$\begin{array}{r} 746_{eight} \\ +\ 552_{eight} \\ \hline \end{array}$$

LATTICE ADDITION

$$\begin{array}{rrr} 1 & 4 & 3_{five} \\ +\ 2 & 3 & 4_{five} \\ \hline \end{array}$$



ABSTRACT ALGORITHM (DO NOT CONVERT TO BASE 10 FIRST!!!):

$$\begin{array}{r} 43_{five} \\ +\ 32_{five} \\ \hline \end{array}$$

$$\begin{array}{r} 32_{five} \\ -\ 13_{five} \\ \hline \end{array}$$

$$\begin{array}{r} 500_{six} \\ -\ 53_{six} \\ \hline \end{array}$$

TAKE HOME PROBLEMS:

ADD USING THE TWO DIFFERENT LEFT-TO-RIGHT ALGORITHMS

$$\begin{array}{r} 9076 \\ +4689 \\ \hline \end{array} \qquad \begin{array}{r} 9\ 0\ 7\ 6 \\ +\ 4\ 6\ 8\ 9 \\ \hline \end{array}$$

ADD USING LATTICE ALGORITHM

$$\begin{array}{r} 5\quad 6\quad 5\quad 9 \\ +\ 7\quad 3\quad 4\quad 8 \\ \hline \end{array}$$

ADD USING SCRATCH ALGORITHM

$$\begin{array}{r} 8\ 7 \\ 6\ 5 \\ +\ 4\ 9 \\ \hline \end{array}$$

ADD USING SCRATCH ADDITION

$$\begin{array}{r} 1\,4\,3_{\text{five}} \\ +2\,3\,4_{\text{five}} \\ \hline \end{array}$$

ADD OR SUBTRACT IN THE GIVEN BASE

$$\begin{array}{r} 3\,2_{\text{five}} \\ +\,1\,3_{\text{five}} \\ \hline \end{array} \qquad \begin{array}{r} 1\,1\,1_{\text{five}} \\ -\ 1\,3_{\text{five}} \\ \hline \end{array}$$