

# T101 SECTION 2-1 NUMERATION SYSTEMS

Do you know what all the following have in common?

IX      ▼▼▼▼▼▼      ●●●●      1001<sub>two</sub>      14<sub>five</sub>      |||||

They are all the number \_\_\_\_\_, but written in various numeration systems or bases.

By comparing our numeration system (Hindu-Arabic or Base-10) with other ancient numeration systems and other bases we hope to gain a clearer picture of how and why we do whole-number computations the way we do.

## I. HINDU-ARABIC (ALSO KNOWN AS BASE-10 OR DECIMAL SYSTEM) (Let's Review)

A. HOW MANY NUMERALS ARE THERE? \_\_\_\_\_

LIST THE NUMERALS: \_\_\_\_\_

B. THE BASE IS \_\_\_\_\_ (This means that each place in any number represents a power of 10 thus each digit in the number represents a certain value, namely, the place value column times the digit's face value)

| Thousands ( $10^3 = 1000$ ) | Hundreds ( $10^2 = 100$ ) | Tens ( $10^1 = 10$ ) | Units ( $10^0 = 1$ ) |
|-----------------------------|---------------------------|----------------------|----------------------|
| 5                           | 3                         | 7                    | 2                    |

For example, in the numeral 5,372, the 5 sits in the place value column of “\_\_\_\_\_” thus the digit 5 represents the value of \_\_\_\_\_ times \_\_\_\_\_ = \_\_\_\_\_.

Similarly,  
The digit 3 represents the value of \_\_\_\_\_ times \_\_\_\_\_ = \_\_\_\_\_.

The digit 7 represents the value of \_\_\_\_\_ times \_\_\_\_\_ = \_\_\_\_\_.

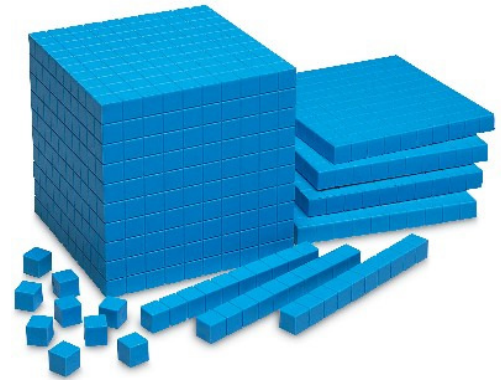
The digit 2 represents the value of \_\_\_\_\_ times \_\_\_\_\_ = \_\_\_\_\_.

Using the powers of 10, write 5,372 in expanded form:

Using the powers of 10, write 12,098 in expanded form:

C. BASE-10 BLOCKS

Base-10 Blocks are used to represent our Base-10 numbers.



1. Use Base-10 blocks to represent the following numbers. Draw your representations below.

234

102

2. Take 18 units, 11 longs, and 2 flats

Trade pieces so you have the fewest possible pieces.  
How many?

Trade pieces so you have the largest number of pieces?  
How many?

## II. EGYPTIAN NUMERATION SYSTEM

This numeration system is based upon tally marks. That is,

|   |   |   |   |   |   |   |   |   |  |
|---|---|---|---|---|---|---|---|---|--|
|   |   |   |   |   |   |   |   |   |  |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |  |

But after 9, they needed a new symbol that began “grouping” to represent certain *sets* of numbers. The next value was ∩ (heel bone) that represented 10.

| Egyptian Numeral | Description     | Hindu-Arabic Equivalent |
|------------------|-----------------|-------------------------|
|                  | Vertical Staff  | 1                       |
| ∩                | Heel Bone       | 10                      |
|                  | Scroll          | 100                     |
|                  | Lotus Flower    | 1,000                   |
|                  | Pointing Finger | 10,000                  |
|                  | Polliwog        | 100,000                 |
|                  | Astonished Man  | 1,000,000               |

The Egyptian system simply followed an *additive property*; that is, the value of the number was the sum of the face values of the numerals.

Convert to Base 10:



Write using Egyptian Numerals: 304,536

### III. ROMAN NUMERATION SYSTEM

| Roman Numeral | Hindu-Arabic |
|---------------|--------------|
|               | 1            |
|               | 5            |
|               | 10           |
|               | 50           |
|               | 100          |
|               | 500          |
|               | 1000         |

Roman Numerals can be combined by using:

1. **Additive property**

$$VI = \quad XVII = \quad CCCXXVIII =$$

2. **Subtractive property**

If a lesser value symbol (only I, X, or C) is to the left of a higher value symbol, the lesser is subtracted from the greater. In general, only one lesser symbol can be to the left of a greater symbol. *This prevents having to write a symbol more than 3 times in a row. NEVER WRITE A SYMBOL MORE THAN 3 TIMES IN A ROW.*

$$IV = \quad XL = \quad CD = \quad CM =$$

$$CXLIV = \quad MCDLXXI =$$

3. **Multiplicative property**

A bar(s) is place of over the symbol(s) indicating you should multiply by 1000 – thus being able to create *very large numbers*

$$\overline{V} = \quad \overline{DCLIX} =$$

$$\overline{\overline{V}} = \quad \overline{\overline{\overline{CVII}}} =$$

#### IV. BASE 5 PLACE VALUE SYSTEM (QUINARY)

The Hindu-Arabic system is a place value system based on 10. This means that the columns are powers of 10. When moving to a different Base system, it simply means that the columns are powers of a different base. It also means that the numerals used will also change.

In a Base 5 system, the base is 5. Therefore the columns are powers of 5.

|       |       |       |       |       |       |               |
|-------|-------|-------|-------|-------|-------|---------------|
| $5^6$ | $5^5$ | $5^4$ | $5^3$ | $5^2$ | $5^1$ | $5^0$         |
|       |       |       |       |       | fives | ones or units |

**WHAT IS THE BASE?** \_\_\_\_\_

**WHAT NUMERALS ARE USED?** \_\_\_\_\_

#### THE MEANING OF A DIGIT

What does the digit 4 mean in the number  $3\ 241_{\text{five}}$ ? \_\_\_\_\_ for a place value of \_\_\_\_\_

What does the digit 2 mean in the number  $3\ 241_{\text{five}}$ ? \_\_\_\_\_ for a place value of \_\_\_\_\_

What does the digit 3 mean in the number  $3\ 241_{\text{five}}$ ? \_\_\_\_\_ for a place value of \_\_\_\_\_

#### WRITING A NUMBER IN EXPANDED FORM

$3\ 241_{\text{five}}$  = \_\_\_\_\_

#### WRITE AS A SINGLE NUMBER IN THE GIVEN BASE.

$(4 \times 5^3) + (1 \times 5^2) + (0 \times 5^1) + (4 \times 5^0) =$  \_\_\_\_\_

$(3 \times 5^7) + (4 \times 5^6) + (2 \times 5^4) + (4 \times 5^3) + (2 \times 5^2) + (3 \times 5^0) =$  \_\_\_\_\_

#### WRITE AS A BASE 10 NUMBER:

$1204_{\text{five}}$

#### WRITE AS BASE 5 NUMBER:

824

#### WRITE THE FIRST 10 COUNTING NUMBERS IN BASE 5:

## V. BASE 2 (ALSO KNOWN AS THE BINARY SYSTEM)

In a Base 2 system, the base is 2. Therefore the columns are powers of 2.

|       |       |       |       |       |       |               |
|-------|-------|-------|-------|-------|-------|---------------|
| $2^6$ | $2^5$ | $2^4$ | $2^3$ | $2^2$ | $2^1$ | $2^0$         |
|       |       |       |       |       | twos  | ones or units |

WHAT IS THE BASE? \_\_\_\_\_

WHAT NUMERALS ARE USED? only \_\_\_\_\_

### THE MEANING OF A DIGIT

What does the first digit of 1 mean in the number  $101_{\text{two}}$ ? \_\_\_\_\_ for a place value of \_\_\_\_\_

What does the second digit of 1 mean in the number  $101_{\text{two}}$ ? \_\_\_\_\_ for a place value of \_\_\_\_\_

What does the digit of 0 mean in the number  $101_{\text{two}}$ ? \_\_\_\_\_ for a place value of \_\_\_\_\_

### WRITING A NUMBER IN EXPANDED FORM

$1011_{\text{two}}$  = \_\_\_\_\_

### WRITE AS A SINGLE NUMBER IN THE GIVEN BASE

$(1 \times 2^4) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) =$  \_\_\_\_\_

### WRITE AS A BASE 10 NUMBER:

$11101_{\text{two}}$

### WRITE AS BASE 2 NUMBER:

69

### WRITE THE FIRST 10 COUNTING NUMBERS IN BASE 2:

## VI. BASE 12 (ALSO KNOWN AS THE DUODECIMAL SYSTEM)

In a Base 12 system, the base is 12. Therefore the columns are powers of 12.

|        |        |        |        |        |         |               |
|--------|--------|--------|--------|--------|---------|---------------|
| $12^6$ | $12^5$ | $12^4$ | $12^3$ | $12^2$ | $12^1$  | $12^0$        |
|        |        |        |        |        | twelves | ones or units |

WHAT IS THE BASE? \_\_\_\_\_

WHAT NUMERALS ARE USED? \_\_\_\_\_

### THE MEANING OF A DIGIT

What does the digit of 9 mean in the number  $96T_{\text{twelve}}$ ? \_\_\_\_\_ for a place value of \_\_\_\_\_

What does the digit of 6 mean in the number  $96T_{\text{twelve}}$ ? \_\_\_\_\_ for a place value of \_\_\_\_\_

What does the digit of T mean in the number  $96T_{\text{twelve}}$ ? \_\_\_\_\_ for a place value of \_\_\_\_\_

### WRITING A NUMBER IN EXPANDED FORM

E  $96T_{\text{twelve}}$  = \_\_\_\_\_

### WRITE AS A SINGLE NUMBER IN THE GIVEN BASE

$(6 \times 12^4) + (10 \times 12^2) + (5 \times 12^1) + (8 \times 12^0) =$  \_\_\_\_\_

### WRITE AS A BASE 10 NUMBER:

$36E_{\text{twelve}}$

### WRITE AS BASE 12 NUMBER:

1, 277

### WRITE THE FIRST 20 COUNTING NUMBERS IN BASE 12:

## VII. EXTENDING TO OTHER BASES

### BASE 8

THE COLUMNS ARE POWERS OF \_\_\_\_\_?

WHAT NUMERALS ARE USED? \_\_\_\_\_

What does the digit 5 mean in the number  $2\ 351_{\text{eight}}$ ? \_\_\_\_\_ for a place value of \_\_\_\_\_

Convert to Base 10:

$$2\ 351_{\text{eight}} = \underline{\hspace{10cm}}$$

$$= \underline{\hspace{10cm}}$$

$$= \underline{\hspace{10cm}}$$

Convert to Base 8:

174

### BASE 3

THE COLUMNS ARE POWERS OF \_\_\_\_\_?

WHAT NUMERALS ARE USED? \_\_\_\_\_

What does the digit 2 mean in the number  $2\ 101_{\text{three}}$ ? \_\_\_\_\_ for a place value of \_\_\_\_\_

Convert to Base 10:

$$2\ 101_{\text{three}} = \underline{\hspace{10cm}}$$

$$= \underline{\hspace{10cm}}$$

$$= \underline{\hspace{10cm}}$$

Convert to Base 3:

46

**TAKE HOME PROBLEMS:**

Convert to Base 10:

DCIV

MCDXLV

CMDXLVII

$\overline{\text{X}}$ MMCDLIV

Write using Roman Numerals:

579

4,709

304,536

8,070

Convert: 68 to base 8

Convert: 503 to base 5

Convert: 240 to base 2