M110 SECTION 9.4 ANNUITIES

An *annuity* is an interest bearing account into which you make a series of payments of the same size. If one payment is made at the *end* of every compounding period, the annuity is called an *ordinary annuity*. The *future value of an annuity* is the amount in the account, *including interest*, after making all the payments.

Example: You begin in January making payments of $100 at the *end* of each month into an account paying 12% yearly interest compounded monthly. How much money will be in the account on May 1st?

Example: You begin in January making payments of $100 at the *end* of each month into an account paying 12% yearly interest compounded monthly. How much money will be in the account on January 1st of the next year?

Since we don't really want to continue the previous example another 8 months, it might be necessary to have a formula to help us. Though the following appears frightful, it is much more simple to use than inching through the months.

<u>FORMULA FOR FUTURE VALUE OF AN ORDINARY ANNUITY</u>

$$A = R \cdot \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}} = R \cdot \left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right] \div \left(\frac{r}{n}\right)$$

R = the regular payment
r = the annual interest rate
t = time in years
n = number of compounding periods per year
A = the future value of the account

So, fill in the following from the above example:

R = ________________
r = ________________
t = ________________
n = ________________
A = the future value of the account

Next, let us use these in the formula and find the future value of the account on January 1st of the following year.

* *Notice that the exponent is the number of regular payments that you make over that time period.*

Example: You begin making payments of $50 at the *end* of each month into an account paying 6% annual interest compounded monthly. How much money will be in the account at the end of 3 years?

Example: You begin making payments of $75 at the *end* of each month into an account paying 6% annual interest compounded monthly. How much money will be in the account after 15 payments?

The next logical question that stems from this idea is: *"If I want to save for a specific amount (say $50,000 for college 18 years from now), how much should my regular payments be?"* The account that you establish for your deposits is called a *sinking fund*. This is just a specific type of annuity, so we do not need another new formula. In this case, we know A ($50,000) and will be solving for R.

<u>FORMULA FOR PAYMENT OF A SINKING FUND</u>

$$R = A \div \frac{\left(1+\frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}} = A \div \left[\left(1+\frac{r}{n}\right)^{nt} - 1\right] \cdot \left(\frac{r}{n}\right)$$

R = the regular payment
r = the annual interest rate
t = time in years
n = number of compounding periods per year
A = the future value of the account

Always round UP to the next higher cent when finding payments (or as directed on MyMathLab)

Example: You want to save $50,000 for your child's college education 18 years from now. You will be making payments at the *end* of each month into an account paying 3.5% annual interest compounded monthly. How much should your monthly payments be?

$$R = A \div \left[\left(1 + \frac{r}{n} \right)^{nt} - 1 \right] \cdot \left(\frac{r}{n} \right)$$

Example: What payments must you make to a sinking fund that pays 9% yearly interest compounded monthly if you want to save $2,500 in two years?

Example: Anna wants to make monthly payments into an annuity that pay 9.6% annual interest to save enough to start a business. If she wants $10,000 in five years, what should her monthly payments be?