Lesson 7.4 (F'11)

Objectives:
1. To find an irrational number.
2. To find the n^{th} root of a number.
3. To estimate square roots.
4. To simplify radicals.

Rational number- a number that can be expressed either as a repeating decimal or as a terminating decimal.

Irrational number- a number that is **not** rational. Their decimal forms are **non-terminating and non-repeating.**

The **Real Numbers** (R) consist of the union of the set of **Rational Numbers** (Q) with the set of Irrational Numbers (S)**.**

To find such decimals (S), we must make sure that the numbers have the following characteristics:

1. There must be an infinite number of nonzero digits to the right of the decimal point.

2. There cannot be a repeating block of digits (no repetend)

Which of the following numbers are irrational?

1. 0.006006006…
2. 0.060060006…
3. 0.01001
4. 0.73173117311173111 1…
5. 0.737373…
6. 0.21060606…

Give an example of an irrational number.

1. 0.23223222322223…
2. 0.02002000200002000002…
3. <u>You Try</u>

Other examples of irrational numbers are:

π - the ratio of the circumference of a circle to its diameter. The value of π has been computed to over a trillion decimal places with no apparent pattern.(we use two <u>**approximations**</u> for pi π 22/7 or 3.14)

e -which is approximately 2.71828 --used in logarithms

Solve:

1. $x – 2 = 0$
2. $x + 2 = 0$
3. $2x – 1 = 0$
4. $x^2 = 2$

Square Roots

Principal Square Root:
If a is any **non-negative** number, the principal square root of a (denoted $\sqrt{a}$) is the **nonnegative** number b, such that $b^2 = a$

$\sqrt{a}$ is a radical $\sqrt{\ }$ is a radical sign a is the radicand

Example: $\sqrt{49} = 7$ $\sqrt{49}$ is the radical
49 is the radicand

Example: $\sqrt{16} =$ ______, $\sqrt{25} =$ ______, $\sqrt{\frac{9}{49}} =$ _____

Find the following: a) The square roots of 64

b) The principal square root of 64

c) $\sqrt{64}$

d) $-\sqrt{64}$

Sometimes the square root of a number is a rational number (like the above example), sometimes it is an irrational number, like the following numbers. $\sqrt{5}, \sqrt{15}, \sqrt{18}$

Whenever *a* cannot be written as b^2, where b is a rational number, then $\sqrt{a}$ is irrational.

Other Roots

Principal Square Root- (even index)

$x^2 = 16$ ___ $x = \sqrt{16}$_____   2 is the index and is understood

$x^4 = 16$_____ $x = \sqrt[4]{16}$ ___   4 is the index

$x^n = b$ $x = \sqrt[n]{b}$ n is the index

$\sqrt{-25}$ = ? $\sqrt[4]{-81}$ = ?

Odd index

$x^3 = 27$______ $x = \sqrt[3]{27}$ ______

$x^5 = -32$______ $x = \sqrt[5]{-32}$ _____

Find:

1. $\sqrt{81}$
2. $\sqrt[3]{-8}$
3. $\sqrt{9+16}$
4. $\sqrt{x^2}$
5. $\sqrt[3]{\dfrac{8}{27}}$

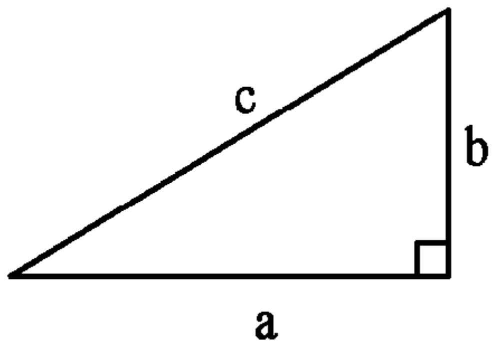
Remember $\sqrt{x^2} = |x|$

Classify these numbers as rational or irrational.

$\sqrt{49}$ $\sqrt{28}$ $\sqrt{121}$ $\sqrt{12}$

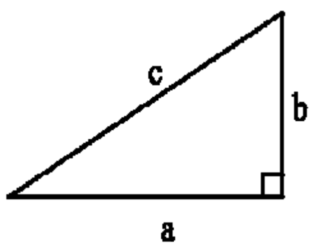
We use square roots in geometry. One example is in the use of the Pythagorean Theorem.

Pythagorean Theorem-- The square of the hypotenuse (c) of a right triangle is equal to the sum of the squares of the other two sides (a and b). $a^2 + b^2 = c^2$

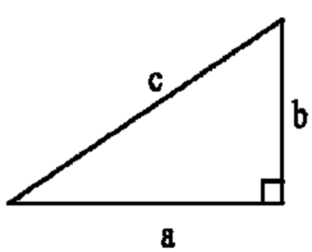


Find the length of the missing side of this triangle.

1. a = 4 b = 6 c = ______



2. a = $\sqrt{3}$ b = ______ c = 5



System of Numbers

REAL NUMBERS

Rational numbers Irrational numbers

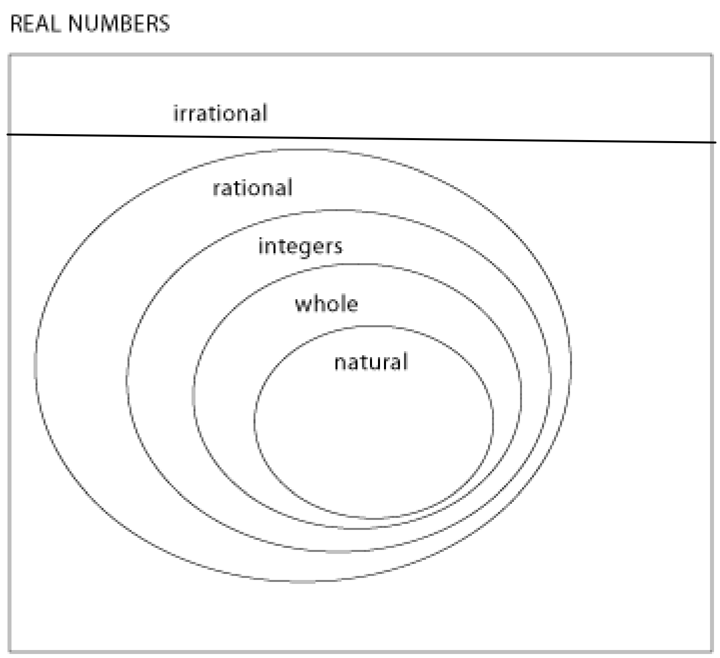
Integers

Whole numbers

Natural numbers

VENN DIAGRAM

NUMBERS



N: Natural Numbers
W: Whole Numbers
I: Integers
Q: Rational Numbers
S: Irrational numbers
R: Real Numbers

Properties page 455--Closure, Commutative, Associative, Identity, inverse, distributive, Denseness

DENSENESS PROPERTY- For real numbers a and c, there exists a real number b such that a < b < c.

<u>Calculator</u>

y^x key

1. $x^{\frac{1}{n}} = \sqrt[n]{x}$
2. $\left(x^m\right)^{\frac{1}{n}} = x^{\frac{m}{n}}$
3. $x^{\frac{m}{n}} = \sqrt[n]{x^m}$
4. $x^{-r} = \frac{1}{x^r}$
5. $(xy)^r = x^r y^r$
6. $\left(\frac{x}{y}\right)^r = \frac{x^r}{y^r}$
7. $(x^r)^s = x^{rs}$

Use your calculator to find each of the following:

1. $\sqrt{4} = \quad 4^{\frac{1}{2}} =$
2. $\sqrt[3]{3} = 3^{\frac{1}{3}} =$
3. $\sqrt{7^3} = 7^{\frac{3}{2}} =$
4. $\left(\sqrt[3]{3}\right)^4 = 3^{\frac{4}{3}} =$

Write each of the following as a radical and then evaluate.

1. $32^{\frac{1}{5}}$
2. $16^{\frac{5}{4}}$
3. $\left(-27\right)^{\frac{1}{3}}$
4. $81^{\frac{-3}{4}}$
5. $\left(-81\right)^{\frac{1}{4}}$

To **simplify** a radical, write the radicand in factored form, using the largest perfect square that is a factor of the radicand. If the radicand has no perfect square factor (other then 1), it is in simplest radical form.

Example: Simplify:

1. $\sqrt{24}$

2. $\sqrt{45}$

3. $\sqrt{200}$

4. $\sqrt{80}$

What if the root is not 2(square)?

5. $\sqrt[5]{64}$

6. $\sqrt[3]{24}$

Try these. Simplify each radical.

1. $\sqrt{12}$ 2. $\sqrt[3]{16}$ 3. $\sqrt{50}$ 4. $\sqrt[4]{243}$

Worksheet

Homework Course Compass Section 7.4 and page 456 #A 4, 7, 13, 15, 18 page 458 #B 15, 16, 17, 18, 20