Lesson 6.3 (F'11)

Objective: 1. To multiply or divide fractions

2. To learn the properties of multiplication and division

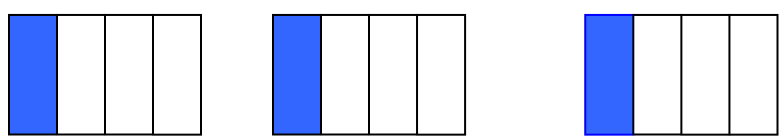
3. To review mental math for fractions

4. To review the properties of exponents and use them with rational numbers

Multiplication

1. Repeated addition. $3\times\frac{1}{4}= \frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}$

b. Area model- A. $3\times\frac{1}{4}$



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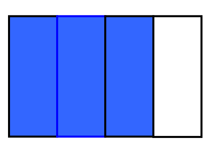
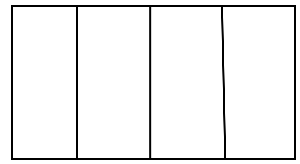


B. But, what happens when both multiplicands are fractions? For example, three-fourths of the students at the local elementary school buy their lunch. Half of these students are girls. What part of the student body are girl lunch buyers? (WE WANT TO FIND "HALF" OF "THREE-FOURTHS" that is, $\frac{1}{2}\times\frac{3}{4}$)

1. Divide the area into 4ths. Shade 3 parts.
   (This is 3/4's of the student body.)



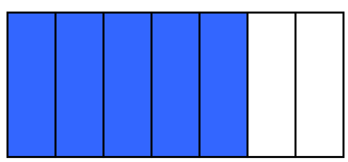
2. Divide the whole area in half. Go the other direction. Shade ½. (This divides the student body in half)

3. Count the overlap of shading.

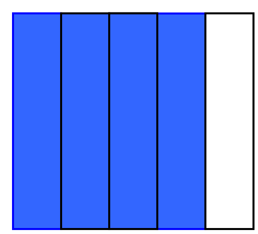
   Thus $\frac{1}{2}\times\frac{3}{4}$ = ________

<u>Try These</u>--Use an area model to find:(Note the problem has been started for you)

b. $\frac{1}{3}\times\frac{5}{7}$



c. $\frac{2}{3}\times\frac{4}{5}$



TRY THESE

Model each of the following with an area model.

1. $\frac{1}{4}$ of $\frac{2}{3}$

2. $\frac{1}{3}\bullet\frac{5}{6}$

3. $\frac{2}{5}\bullet\frac{3}{4}$

These examples lead us to the:

Definition of Multiplication of Rational Numbers

If $\frac{a}{b}$ and $\frac{c}{d}$ are any rational numbers, then their product is given by $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$. (THE ALGORITHM)

Try This--Find each product.

1. $\frac{5}{8} \bullet \frac{2}{3}$

2. $\frac{56}{88} \bullet \frac{-4}{7}$

3. $3\frac{1}{2} \bullet 7\frac{2}{3}$ (2 ways)

4. If $\frac{3}{4}$ of the population of a certain city are college graduates and $\frac{2}{3}$ of the college graduates are women, what fraction of the population are female college graduates?

Properties of Rational Number Multiplication

Given any rational numbers, $\frac{a}{b}$, $\frac{c}{d}$, and $\frac{e}{f}$

1. Closure
2. Commutative
3. Associative
4. **Multiplicative Identity of Rational Numbers**
   The number ________ is the unique number such that for every rational number $\frac{a}{b}$, $\frac{a}{b}\cdot 1 = 1\cdot\frac{a}{b} = \frac{a}{b}$

5. **Multiplicative Inverse of Rational Numbers**
   For any nonzero rational number $\frac{a}{b}$, $\frac{b}{a}$ is the unique rational number such that $\frac{a}{b}\cdot\frac{b}{a} = 1 = \frac{b}{a}\cdot\frac{a}{b}$
   (The multiplicative inverse of $\frac{a}{b}$ is also called the ***reciprocal*** of $\frac{a}{b}$.)

6. **Distributive Property of Multiplication over Addition for Rational Numbers**
   If $\frac{a}{b}$, $\frac{c}{d}$, $\frac{e}{f}$ are any rational numbers, then $\frac{a}{b}\left(\frac{c}{d} + \frac{e}{f}\right) =$ ______

7. **Multiplication Property of Equality for Rational Numbers**
   If $\frac{a}{b}$ and $\frac{c}{d}$ are any rational numbers such that $\frac{a}{b} = \frac{c}{d}$, and $\frac{e}{f}$ is any rational number, then

$$\frac{a}{b}\cdot\frac{e}{f} = \frac{c}{d}\cdot\frac{e}{f}$$

8.Multiplication Property of Zero for Rational Numbers

If $\frac{a}{b}$ is any rational number, then $\frac{a}{b}\cdot 0 = 0\cdot\frac{a}{b} = 0$

9. Multiplication Property of Inequality for Rational Numbers.

If $\frac{a}{b} > \frac{c}{d}$ and $\frac{e}{f} > 0$ then $\frac{a}{b} \bullet \frac{e}{f} > \frac{c}{d} \bullet \frac{e}{f}$

If $\frac{a}{b} > \frac{c}{d}$ and $\frac{e}{f} < 0$ then $\frac{a}{b} \bullet \frac{e}{f} < \frac{c}{d} \bullet \frac{e}{f}$

Find the multiplicative inverse (reciprocal) **<u>and</u>** the additive inverse of each number.

1. $-\frac{3}{4}$

M____A____

2. $2\frac{1}{3}$

M____A____

3. 0

M____A____

4. -6

M____A____

<u>Try This</u>--SOLVE

1. $\frac{3}{4}x = 12$

2. $-\frac{3}{5}x = 90$

3. Three-fourths of the students at IUX commute more than 10 miles to school each day. If 900 students at the college commute each day, how many students attend IUX?

4. $\frac{2}{3}x+\frac{3}{4}x=2$

5. $\frac{x}{c}+\frac{x}{d}=1$ Solve for x in terms of c and d.

Division

Principles and Standards   page 388

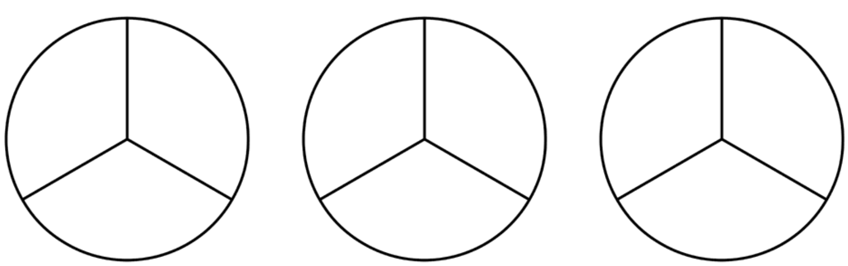
Division of Rational Numbers

In the *Principles and Standards*, we find the following statement concerning division o rational numbers:

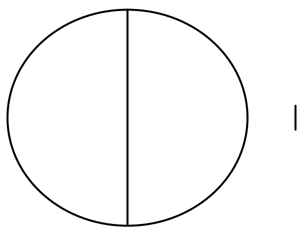
> The division of fractions has traditionally been quite vexing for students. Although "invert and multiply" has been a staple of conventional mathematics instruction and although it seems to be a simple way to remember how to divide fractions, students have for a long time had difficulty doing so. Some students forget which number is to be inverted, and others are confused about when it is appropriate to apply the procedure. A common way of formally justifying the "invert and multiply" procedure is to use sophisticated arguments involving the manipulation of algebraic rational expressions—arguments beyond the reach of many middle-grades students. This process can seem very remote and mysterious to many students. Lacking an understanding of the underlying rationale, many students are therefore unable to repair their errors and clear up their confusions about division of fractions on their own. An alternative approach involves helping students understand the division of fractions by building on what they know about the division of whole numbers. (p. 219)

Remember $8 \div 2$ "means how many 2's are in 8"

1. **Area Model**: a. $3 \div \frac{1}{3}$ "means how many $\frac{1}{3}$'s are in 3"



b. $\frac{1}{2} \div \frac{1}{4}$ “means how many $\frac{1}{4}$’s are in $\frac{1}{2}$



Number Line Model

Steve has 3 1/2 feet of dowel rod. He must cut this into 1/2 foot pieces, how many 1/2 feet pieces are possible?

←------|--------|--------|--------|--------|--------|--------|--------|----→

TRY THESE—Use an area model to divide. Make sure you can explain your answer.

1. $\frac{3}{4} \div \frac{1}{8}$

2. $3 \div \frac{1}{2}$

These examples lead us to the:

Definition of Division of Rational Numbers

If $\frac{a}{b}$ and $\frac{c}{d}$ are any rational numbers and $\frac{c}{d}$ is not zero, then $\frac{a}{b} \div \frac{c}{d} = \frac{e}{f}$ if, and only if, $\frac{e}{f}$ is the unique rational number such that $\frac{c}{d} \bullet \frac{e}{f} = \frac{a}{b}$.

Example

1. $\frac{3}{5} \div \frac{2}{3} = x$ means $(\frac{2}{3}x) = \frac{3}{5}$ next multiply each side by $\frac{3}{2}$.

Then $(\frac{2}{3}x)(\frac{3}{2}) = \frac{3}{5}(\frac{3}{2})$ and $x = \frac{3}{5} \div \frac{2}{3}$, therefore $\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \cdot \frac{3}{2}$

This brings us to our --

Algorithm for Division of Fractions

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$, where $\frac{c}{d} \neq 0$

2. $2\frac{1}{3} \div 4\frac{2}{3}$

2. Cookies are sold in packages, each weighing $3\frac{1}{2}$ ounces. If there is a supply of $18\frac{2}{3}$ ounces of cookies, how many packages of cookies can be made? How many ounces of cookies will be left?

MENTAL MATHEMATICS AND ESTIMATION

MENTAL MATHEMATICS

(Use mental mathematics to find exact answer.)

$(36 \cdot 25) \cdot \frac{1}{9}$ $\quad 5\frac{1}{6} \cdot 12$ $\quad \dfrac{6\frac{1}{2}}{\frac{1}{2}}$ $\quad \frac{4}{5} \cdot 20$

Estimation

(Get an answer that close)

1. $3\frac{1}{4} \bullet 7\frac{8}{9}$

2. $24\frac{3}{8} \div 4\frac{1}{8}$

Exponents

Definition of a to the m*th* Power

$\underbrace{a^m = a \bullet a \bullet a \bullet a ... \bullet a}_{\text{m factors}}$ where a is any rational number and m is any natural number.

1. $x^3 \bullet x^2 =$

2. $\frac{x^3}{x^2} =$

3. $\frac{x^3}{x^3} = x^0 = 1$

4. $\frac{x^2}{x^5} =$

Properties

For any nonzero rational number *a* and any integers *m* and *n*:

1. $a^m \bullet a^n = a^{m+n}$

2. $\frac{a^m}{a^n} = a^{m-n}$

3. $(a^m)^n = a^{mn}$

4. $(\frac{a}{b})^m = \frac{a^m}{b^m}$

5. $a^0 = 1$

6. $a^{-n} = \frac{1}{a^n}$

7. $(ab)^n = a^n \bullet b^n$

8. $(\frac{a}{b})^{-m} = (\frac{b}{a})^m$

EXAMPLE

Simplify-leave answers in simplest form using positive exponents in the final answer.

1. $8^6 \bullet 2^{-3}$

2. $\left(\frac{1}{2}\right)^9 \div \left(\frac{1}{2}\right)^6$

3. $\left(\frac{a^2}{b}\right)^7$

4. $(x^2y^{-2})^{-3}$

Use properties of exponents to justify:

1. $(-a)^2 \neq -a^2$

2. $(x^{-2}y^{-2})^{-2} = x^4y^4$

Solve

1. $x^2 = 64$

2. $3^x = 9^2$

3. $9^x = 27$

4. $3^x \bullet 3^4 = 3^8$

Class work
Homework Course Compass Section 6.3 and page 399 A#1, 2, 4, 13, 14, 21 page 400 B #1, 3, 4, 12, 15, 17, 19, 20, 22, 24, 26, 28