Lesson 6.1 (F'11)

OBJECTIVES:
1. To understand fractions
2. To reduce fractions
3. To compare fractions
4. To find fractions between two given fractions

Syllabus quiz on Course Compass (My Math Lab)

FRACTIONS

Beyond understanding whole numbers, young children can be encouraged to understand and represent commonly used fractions in context, such as 1/2 of a cookie or 1/8 of a pizza, and to see fractions as part of a whole unit or of a collection. Teachers should help students develop an understanding of fractions as division of numbers. And in the middle grades, in part as a basis for their work with proportionality, students need to solidify their understanding of fractions as number.(Page 33 *Principle and Standards*)

In Grades K-2 children should understand and represent commonly used fractions such as 1/4, 1/3, and 1/2.

In grades 3-5, children should develop an understanding of fractions to include them as parts of unit wholes, as parts of a collection, as locations on a number line, and as divisions of whole numbers. Children should use models, benchmarks, and equivalent forms to judge the size of fractions. They should be able to develop and use strategies to estimate computations involving fractions. (page 392 *Principle and Standards*).

Need for fractions

Solve:
1. $x + 2 = 5$
2. $x + 4 = 3$
3. $6x = 3$

A **fraction** is an ordered pair of numbers a and b **(not necessarily integers)**, $b \neq 0$, written $\frac{a}{b}$ or a/b.

The number **a** is called the **numerator** of the fraction, and the number **b** is called the **denominator** of the fraction.

If **a** and **b** are integers, we call the fraction a **rational numbers (Q)**. **Q** = $\{\frac{a}{b} |$ a and b are integers, $b \neq 0\}$

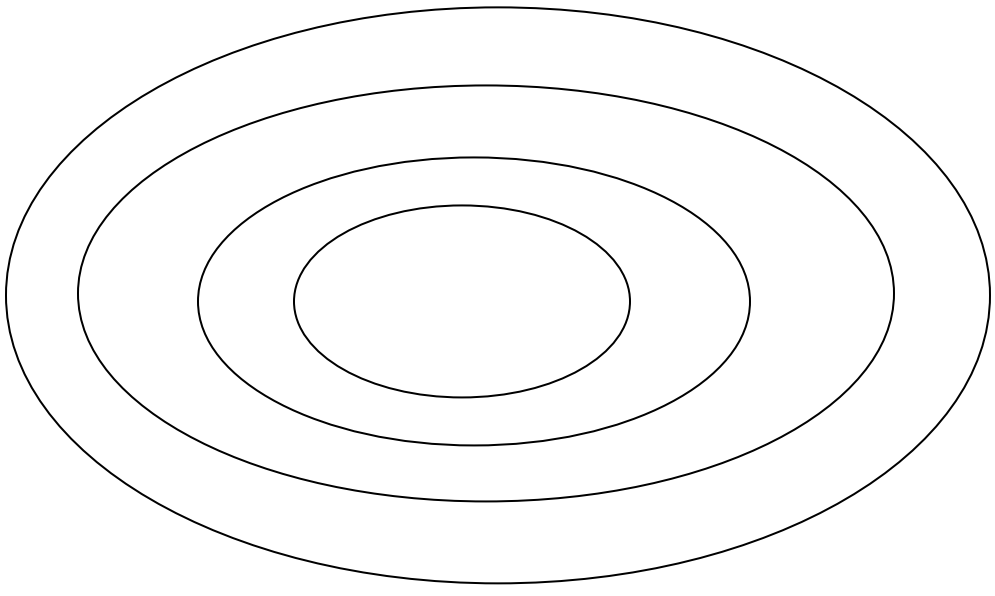
Example: $\frac{x+2}{y-3}$, $\frac{3}{7}$, $\frac{2z}{5}$, $\frac{-4}{9}$, $\frac{\sqrt{3}}{2}$, $\frac{0}{5}$

<u>TRY THESE:</u> Classify each of the expressions above as rational numbers, fractions or both? Remember any integer x is view as $\frac{x}{1}$ and therefore is a rational number.

Now fill in the following Venn Diagram

VENN DIAGRAM: OF THE REAL NUMBERS

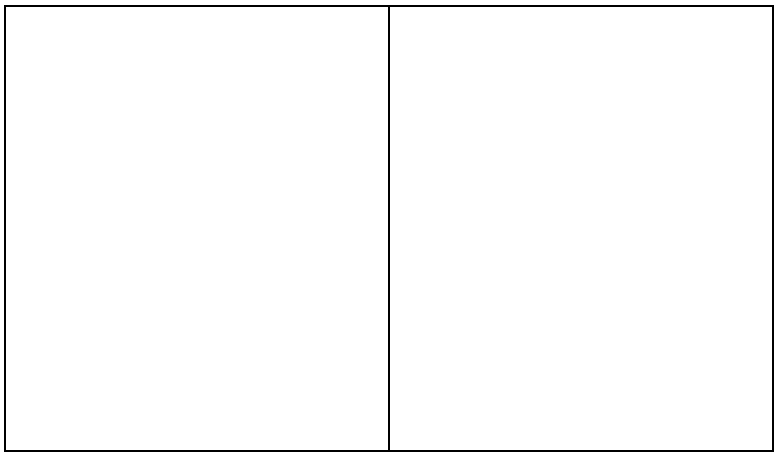
N: Natural Numbers
W: Whole Numbers
I: Integers
Q: Rational Numbers
R: Real Numbers



TRUE OR FALSE:

$I \subseteq Q$ $W \subseteq I$ $I \subseteq N$ $N \subseteq W$ $N \subseteq Q$ $W \subseteq Q$

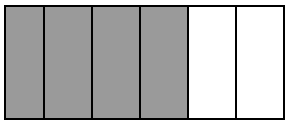
TRY THESE: A VCR has an SP setting and an EP setting. The SP setting allows 2 hours of recording, and the EP setting allows 6 hours of recording. Amy taped a half-hour show using the SP setting on a tape and then switched to the EP setting. How many more 30-minute show can she tape on the same tape and in this new setting.

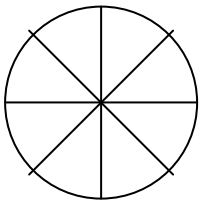


MODELS FOR FRACTIONS

1. **Colored Regions Model**

http://www.visualfractions.com/EnterCircle.html

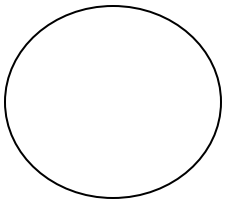




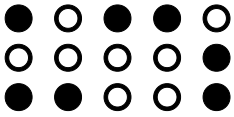
What fraction is shaded? ____ What fraction is shaded?_____

1. Shade $\frac{2}{5}$

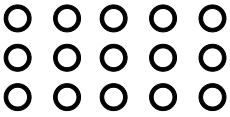
2. Shade $\frac{3}{8}$



2. **Set Model**



What fraction of the balls are shaded?



Shade $\frac{2}{5}$ of the balls.

3. **Number Line Model**

Shade $\frac{5}{6}$ of the unit segment

|--------|--------|--------|--------|--------|--------|
0 1

TRY THESE:

Draw a number line and include the following points.

$\frac{-5}{4}, -\frac{3}{4}, -\frac{1}{4}, 0, \frac{3}{4}, \frac{5}{4}$

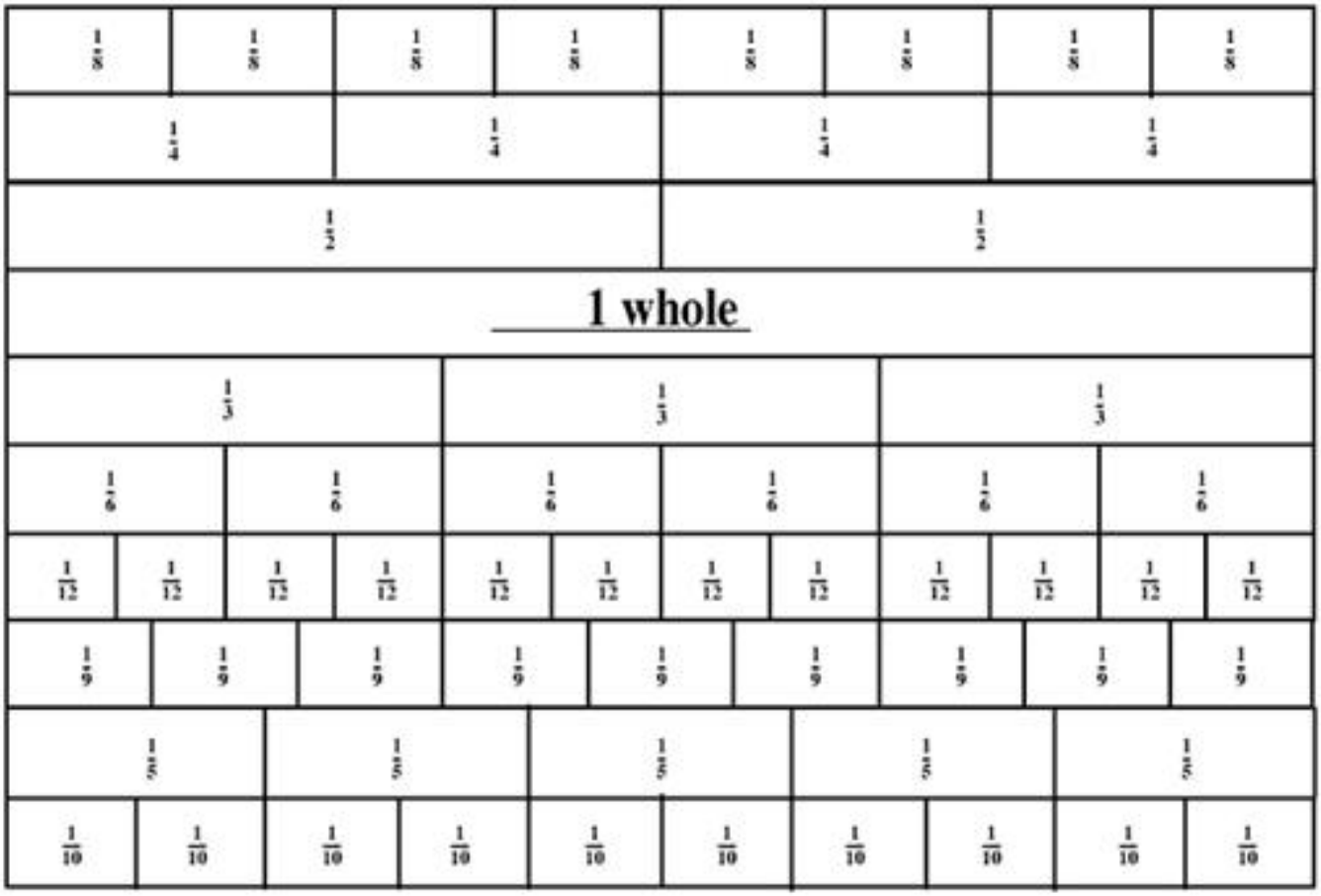
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

4. **Fraction Strip Model**

When students are asked to show an understanding of the concept of a fraction, the **area model** is preferred over the set model because the total area is more flexible.

Fraction strips can be folded into parts so that the resulting strip is equal in length to the given fraction. Folds may be made ONLY on the lines of the strips.

$\frac{1}{2} = \_ = \_$ $\frac{6}{8} = \_ = \_$



Interesting note: The Chinese read $\frac{1}{3}$ as “out of three parts, (take) one.

Proper fraction- A fraction $\frac{a}{b}$, where $0 \le |a| < |b|$.

A proper fraction is a fraction whose absolute value is greater than or equal to 0 but less than 1.

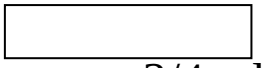
Examples of proper fractions $\frac{1}{2}, \frac{7}{8}, \frac{12}{17}$

Improper fraction-A fraction $\frac{a}{b}$, where $0 < |b| \le |a|$

An improper fraction is a fraction whose absolute value is greater than or equal to 1.

Examples of improper fractions $\frac{3}{2}, \frac{11}{10}, \frac{29}{3}, \frac{5}{5}$

TRY THESE:

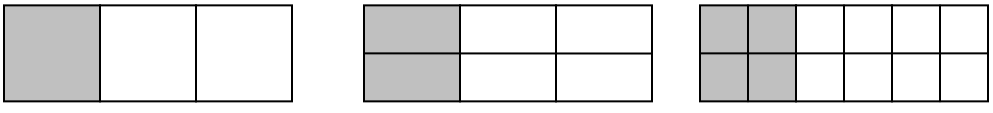


If the above rectangle represent 3/4 , draw rectangles for 1, 1/4, 1/2, 3/2, and 4/3.

Equivalent Fractions

Paper Folding $\frac{1}{3}$ Fold in half $\frac{2}{6}$ Fold in half $\frac{4}{12}$

$\frac{1}{3}$ $\frac{2}{6}$ $\frac{4}{12}$ all represent the same rational number and are equivalent fractions or equal fractions.



$\frac{1}{3}$ $\frac{2}{6}$ $\frac{4}{12}$

Notice that the value of the fraction does not change when the numerator and denominator are multiplied by the same number.

TRY THESE: Name four rational numbers equivalent to $\frac{1}{4}$.

Remember: Fractions that express the same quantity are called **equivalent fractions.**

Fundamental Law of Fractions

Let $\frac{a}{b}$ be any fraction and n be a nonzero integer, then $\frac{a}{b} = \frac{an}{bn}$.

Actually, n can be any nonzero number. If we let n = -1, then:

$$\frac{-a(-1)}{b(-1)} = \frac{a}{-b}$$

We can use this law to "build up" fractions and also to simplify "reduce" them.

SIMPLIFY:

1. $\frac{70}{210}$

2. $\frac{35xy^2}{49x^2y}$

Definition of Simplest Form - (lowest terms)

A rational number $\frac{a}{b}$ is in simplest form if **a** and **b** have no common factors greater than 1, that is a and b are **relatively prime**. (that is, the GCD $(a, b) = 1$).

TRY THESE: Write each of the following in simplest form.

1. $\frac{84}{105}$

2. $\frac{25a^2b}{30ab^2}$

3. $\frac{(a+b)^2}{3a+3b}$

4. $\frac{x^2+x}{x+1}$

5. $\frac{3+x^2}{3x^2}$

6. $\frac{3+3x^2}{3x^2}$

7. $\frac{x^2-y^2}{x+y}$

8. $\frac{x+4}{2}$

Equality of Fractions

Methods to determine if two fractions are equal

A. **Reduce to simplest form.**

$\frac{8}{12} =$ and $\frac{14}{21} =$

B. **Rewrite both fractions with the same least common denominator**.

$\frac{8}{12} =$ and $\frac{14}{21} =$

What is the LCD??

Rewrite each fraction with this denominator.

C. **Write both fractions with any common denominator**

Rewrite each fraction with a denominator of 12 x 21.

$\frac{8}{12} =$ and $\frac{14}{21} =$

Let's use method 3 to explain our next Property.

When are $\frac{a}{b}$ and $\frac{c}{d}$ equivalent?

$\frac{a}{b} = \frac{a \bullet d}{b \bullet d} = \frac{ad}{bd}$ and $\frac{c}{d} = \frac{c \bullet b}{d \bullet b} = \frac{cb}{db}$

therefore $\frac{ad}{bd} = \frac{cb}{db}$ thus

$\frac{a}{b} = \frac{c}{d}$ if and only if ad = bc.

Property

Two fractions $\frac{a}{b} and \frac{c}{d}$ are equal if, and only if, ad = bc.

TRY THESE: Use the above property to determine if the following fractions are equal:

$\frac{14}{63}$ and $\frac{10}{45}$ $\frac{17}{27}$ and $\frac{25}{45}$

Ordering Fractions

Methods to order fractions

A. **Like Denominators:** It is simple to compare fractions that have like denominators; just compare the numerators. The one with the greater numerator is the greater fraction.

Theorem 5.1 If a, b, and c are integers and b > 0, $\frac{a}{b} > \frac{c}{b}$ if and only if, a > c.

Is this theorem true if b < 0? WHY?

TRY THESE: Insert <, >, or =.

$\frac{5}{12}$ ___ $\frac{7}{12}$ $\frac{13}{22}$ ___ $\frac{9}{22}$

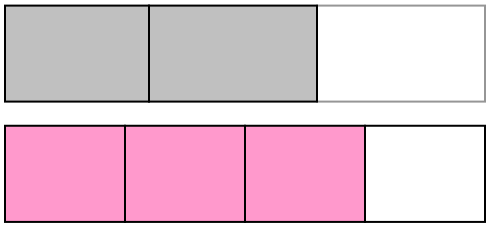
What if the denominators are not equal?

B. **Unlike Denominators**:

1. **Area Model or Fraction Strips Model**

Compare

$\frac{2}{3} and \frac{3}{4}$



2. **Getting the LCD** and comparing

$\frac{2}{3} and \frac{3}{4}$

3. **Using Cross Products**

THEOREM 5.2 If *a*, *b*, *c*, and *d*, are integers and $b > 0$, $d > 0$, then $\frac{a}{b} > \frac{c}{d}$ if, and only if, $ad > bc$.

Therefore, $\frac{3}{7} > \frac{5}{12}$ because $3(12) > 5(7)$.

$\frac{3}{4} > \frac{9}{13}$ because $3(13) > 4(9)$

$\frac{15}{16} > \frac{8}{9}$ because 15(9) > 16(8)

$\frac{-1}{2} > \frac{5}{-8}$

First we must place the negative signs in the numerator thus $\frac{-1}{2} > \frac{5}{-8}$ becomes $\frac{-1}{2} > \frac{-5}{8}$ because -8 > -10

TRY THESE: $\frac{3}{8}$ and $\frac{5}{11}$ $\frac{-9}{12}$ and $\frac{4}{-5}$

TRY THESE Arrange each of the following in ascending order. **Do not convert the fractions to decimals. Show work to justify your answer. Try #4 for next class.**

1. $\frac{7}{8}, \frac{9}{11}$

2. $\frac{17}{32}, \frac{19}{40}$

3. $\frac{11}{17}, \frac{13}{17}, \frac{12}{17}$

4. $\frac{3}{11}, \frac{7}{23}, \frac{2}{9}, \frac{5}{18}$

Denseness of Rational Numbers

Property -Given rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, there is a rational number between these two numbers

1. Find a fraction between $\frac{1}{3}$ and $\frac{1}{2}$.

   Rewrite with a common denominator.

2. Find a fraction between $\frac{7}{8}$ and $\frac{5}{6}$. How many rational numbers lie between $\frac{7}{8}$ and $\frac{5}{6}$?

<u>TRY THESE:</u> Find three fractions between $\frac{1}{4}$ and $\frac{1}{3}$.

Class work Fraction Circle Worksheet
Homework Course Compass section 6.1 and problems from the book page 357 #A 5, 6, 11, 22 page 359# B 3, 6, 8, 11, 18, 19, 23, 24