Lesson 6.2 (F'11)

OBJECTIVES: 1. To add and subtract fractions.

2. To learn the properties of addition and subtraction.

3. To estimate with rational numbers.

ADDITION

1. Region Model- $\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$

2. Number Line Model-

←------|--------|--------|--------|--------|--------|--------|--------|----→

Definition of Rational Number Addition

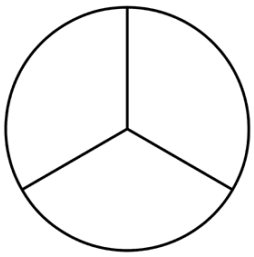
If $\frac{a}{b}$ and $\frac{c}{b}$ are rational numbers, then $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$.

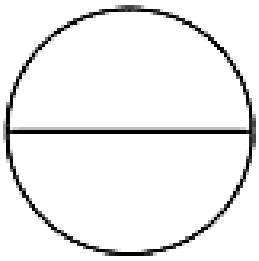
What if the denominators are different?

Use the area model to add:

$$\frac{1}{3} + \frac{1}{2}$$

Let's use Polya's four-step process. 1. Understand the problem.





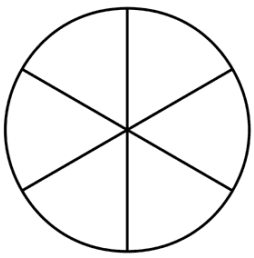
2. Devise a plan. But, how do we accomplish this? We need to find a way to combine the two drawings to find the sum. We need each circle to have the same number of pieces.

3. Carry out the plan. Let's "build-up" each fraction:

$\frac{1}{3} = \_ = \_ = \_$ $\frac{1}{2} = \_ = \_ = \_$

What is the common denominator?_____

Now let's divide our circles into this many pieces and shade each circle.



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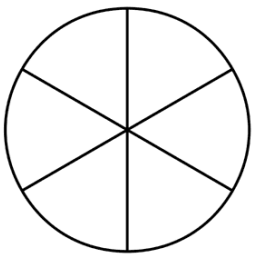

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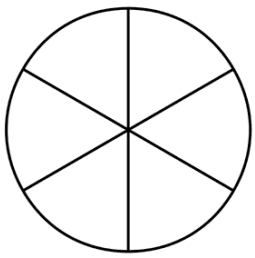


3. Looking back.

4.

<u>TRY THESE--</u>Model each of the following with an <u>area or region model</u>. Make sure you can explain your answer.

1. $\frac{1}{2}+\frac{1}{3}$ 2. $\frac{1}{3}+\frac{5}{12}$

Property: If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$.

Examples—Add each of the following using the LCD

1. $\frac{5}{9}+\frac{2}{15}$

2. $\frac{7}{15}+\frac{5}{12}$

3. $\frac{4}{9}+\frac{1}{-3}$

4. $\frac{3}{x}+\frac{2}{y}$

Mixed Number- Numbers that are made up of an integer and a fractional part of an integer. $a\frac{b}{c} = a + \frac{b}{c}$

Example: $3\frac{1}{3} = 3 + \frac{1}{3}$ $-4\frac{3}{4} = -(4 + \frac{3}{4})$

"In a NAEP test, students were given the following problem:

$5\frac{1}{4}$ is the same as: a. $5 + \frac{1}{4}$ b. $5 - \frac{1}{4}$ c. $5 \times \frac{1}{4}$ or d. $5 \div \frac{1}{4}$

Only 47% of the seventh graders answered correctly.

TRY THIS--what is wrong with this student's problem.

$$-4\frac{3}{4} = \frac{(-4x4)+3}{4} = \frac{-16+3}{4} = \frac{-13}{4}$$

Change each mixed number to an improper fraction.

1. Using the Conventional Algorithm

$$6\frac{1}{2} = \frac{6 \bullet 2+1}{2} = \frac{13}{2}$$

Change with Meaning

$$6 + \frac{1}{2} = \frac{12}{2}+\frac{1}{2}=\frac{13}{2}$$

(You do the next two)

Using the Conventional Algorithm

Change with Meaning

2. $2\frac{4}{9} =$

Using the Conventional Algorithm

Change with Meaning)

3. $-4\frac{1}{5}$

Example:

Change each improper fraction to a mixed number.

Using the Conventional Algorithm

Change with Meaning

1. $\frac{37}{10} =$

$$\frac{37}{10}=\frac{10}{10}+\frac{10}{10}+\frac{10}{10}+\frac{7}{10}=3\frac{7}{10}$$

Try These.

1. $\frac{37}{8}$ 2. $\frac{29}{5}$

Calculator

1. Add: $3\frac{11}{15}+5\frac{5}{21}$

2. Change to an improper fraction: $156\ \frac{33}{34}$

3. Change to a mixed number: $\frac{2347}{124}$

Properties of Addition for Rational Numbers (Q)

Additive inverse property of rational numbers. For any rational number $\frac{a}{b}$, there exists a unique number $-\frac{a}{b}$, the additive inverse of $\frac{a}{b}$, such that $\frac{a}{b}+(-\frac{a}{b})=0=(-\frac{a}{b})+\frac{a}{b}$.

Find the additive inverse of each fraction.

1. $\frac{4}{9}$ 2. $\frac{-3}{10}$ 3. $\frac{x+y}{z}$ 4. $-4\frac{1}{3}$

Addition property of equality. If $\frac{a}{b}$ and $\frac{c}{d}$ are any rational numbers such that $\frac{a}{b} = \frac{c}{d}$ and if $\frac{e}{f}$ is any rational number then, $\frac{a}{b} + \frac{e}{f} = \frac{c}{d} + \frac{e}{f}$.

TRY THIS--SOLVE: $x - \frac{1}{4} = \frac{3}{8}$

PROPERTIES OF ADDITION FOR RATIONAL NUMBERS

Given any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ where, b,d and f are non-zero integers:

1. Closure

2. Commutative

3. Associative

4. Additive Identity

ADDITION OF MIXED NUMBERS

(Know how to add using the given mixed numbers)

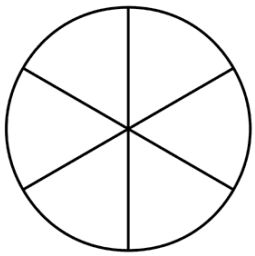
$2\frac{3}{8}+5\frac{5}{6}$

$$\begin{array}{r} 2\frac{3}{8} \\ +\,5\frac{5}{6} \\ \hline \end{array}$$

$82\frac{9}{25}+17\frac{33}{50}$

SUBTRACTION

1. Take away model: $\frac{5}{6}-\frac{3}{6}$



2. Number line model: $\frac{5}{6}-\frac{3}{6}$

←------|--------|--------|--------|--------|--------|--------|--------|----→

3. Missing addend model: $\frac{5}{6}-\frac{3}{6}$ means $\frac{3}{6}+\_\_\_=\frac{5}{6}$.

Definition of Subtraction of Rational Numbers Terms of Addition

If $\frac{a}{b}$ and $\frac{c}{d}$ are any rational numbers, then $\frac{a}{b}-\frac{c}{d}=x$ if, and only if, $\frac{c}{d}+x=\frac{a}{b}$.

Theorem 5.4

If $\frac{a}{b}$ and $\frac{c}{d}$ are any rational numbers, then $\frac{a}{b}-\frac{c}{d}=\frac{a}{b}+(\frac{-c}{d})$.

Let's find each difference (without a calculator).

1. $\frac{4}{5}-\frac{2}{3}$

2. $\frac{y}{2}-\frac{y}{3}$

2. $\frac{2}{x-y}-\frac{3}{x+y}$

4. $\frac{2-x}{6-3x}+\frac{4-2x}{3x-6}$

5. $5\frac{1}{3}-2\frac{3}{4}$ Use 2 methods

I

$$5\frac{1}{3}$$
$$-\ 2\frac{3}{4}$$

II

$$5\frac{1}{3}-2\frac{3}{4}$$
$$\frac{16}{3}-\frac{11}{4}$$

TRY THESE

Use Method I to subtract. (Know how to subtract using the given mixed numbers)

1. $15\frac{1}{2} - 10\frac{7}{8}$ 2. $9\frac{1}{6} - 2\frac{2}{3}$ 3. $147\frac{1}{5} - 40\frac{1}{2}$

Definition of Greater Than or Less Than in Terms of Subtraction

$\frac{a}{b} < \frac{c}{d}$ if $\frac{c}{d} - \frac{a}{b} > 0$

$\frac{c}{d} > \frac{a}{b}$ iff $\frac{a}{b} < \frac{c}{d}$

Verify the following using the above definition.

1. $\frac{3}{4} + \frac{4}{3} > 2$ this is ture if $(\frac{3}{4} + \frac{4}{3})$ - 2 > 0

$\frac{3}{4} + \frac{4}{3} - 2 = \frac{9+16-24}{12} = 1/12 > 0$, hence $\frac{3}{4} + \frac{4}{3} > 2$

Estimation with rational numbers

"Estimation helps us to make practical decisions in our everyday life."(Book pg 373)

Many times when estimating with fractions, it is helpful to round to a *convenient fraction* – for instance

$0, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{2}{3}, \frac{3}{4},$ or 1.

For example, if you got 59 out of 80 questions correct on your test, this is about $\frac{60}{80}$ or $\frac{6}{8}$ or $\frac{3}{4}$Then we can conclude that $\frac{3}{4}$ is a HIGH ESTIMATE. (You actually got less than $\frac{3}{4}$ of the test correct, since 59 < 60, then $\frac{59}{80} < \frac{60}{80} = \frac{3}{4}$).

TRY THESE--Estimate:

1. $\frac{12}{13}+\frac{7}{8}$ 24% 13yr olds

2. $15\frac{1}{4}+7\frac{3}{5}$

3. $\frac{217}{413}+\frac{698}{703}$

4. $6\frac{7}{8}-\frac{7}{12}$

5. A student added $\frac{3}{4}$ and $\frac{1}{3}$ and obtained $\frac{4}{7}$. How could you use estimation to show the student that his answer could not be correct?

6. Use Front end estimation to add

$$1\frac{1}{9}+4\frac{3}{8}+6\frac{7}{8}+\frac{4}{5}$$

TRY THESE

1. Round each rational number to the nearest whole number.

$$\frac{2}{5} \qquad 4\frac{3}{7} \qquad \frac{8}{3} \qquad 7\frac{3}{6}$$

2. Replace each rational number by a close approximation that is in simpler from.

Use $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{3}$, $\frac{3}{4}$

$$4\frac{17}{30} \qquad 12\frac{5}{26} \qquad \frac{4}{13} \qquad \frac{19}{80}$$

Class work
Home work Course Compass Section 6.2 and page 375
A # 4, 5, 6, 11, 15 and page 377 #4, 15, 16