Lesson 4.3(F"11)

Objectives: Determine if two functions are inverses.
Find the inverse of a function.
Determine if a function is one-to-one.

EXAMPLE: Let f(x) = 3x + 2 and g(x) = $\frac{x-2}{3}$.

Find: f(0) and g(2) f(2) and g(8)

f(-4) and g(-10) f(g(x)) and g(f(x)).

These are inverse functions.

Inverse Functions

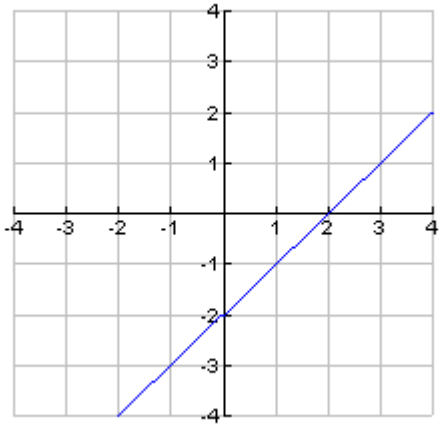
Functions *f* and *g* for which $f(g(x)) = x$ for all *x* in the domain of *g*, **and** $g(f(x)) = x$ for all *x* in the domain of *f*, are called inverse functions. f^{-1} is read as "*f* inverse".

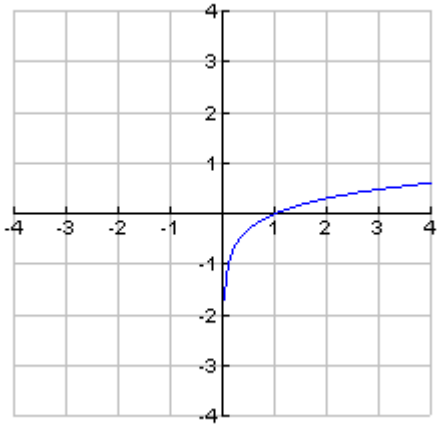
If *f* and *g* are inverse functions then whenever the pair *(a,b)* satisfies *y = f(x)*, then the pair *(b,a)* satisfies *y = g(x).*

EXAMPLE; Find the inverse of the following set of ordered pairs.

{(0,1), (2, 3), (3, 4), (4, 5)}

Given the graph of f below, draw the graph of f^{-1}.





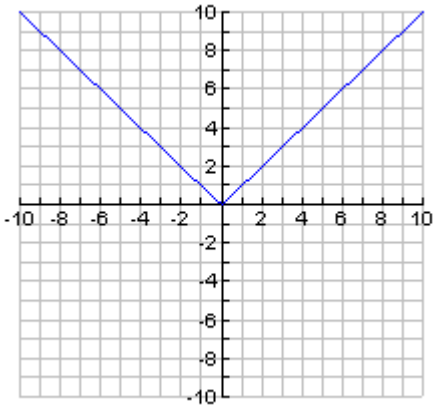
One-to-One Function

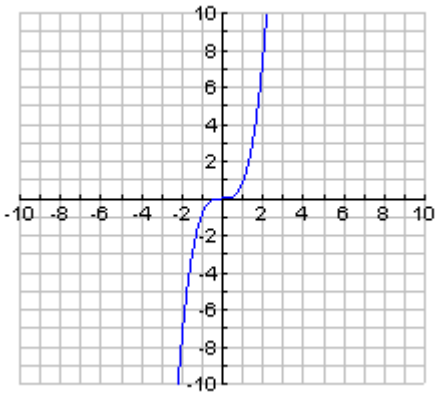
A one-to-one function has **exactly one output** for each input and **exactly one input** for each output. This means there is a one-to-one correspondence between the independent and dependent variables defining the function. Every one-to-one function has an inverse function.

Graphically – Horizontal Line Test

A function is one-to-one if no horizontal line can intersect the graph of the function in more than one point.

EXAMPLE: Are the following one-to-one?





Finding the Inverse of a Function

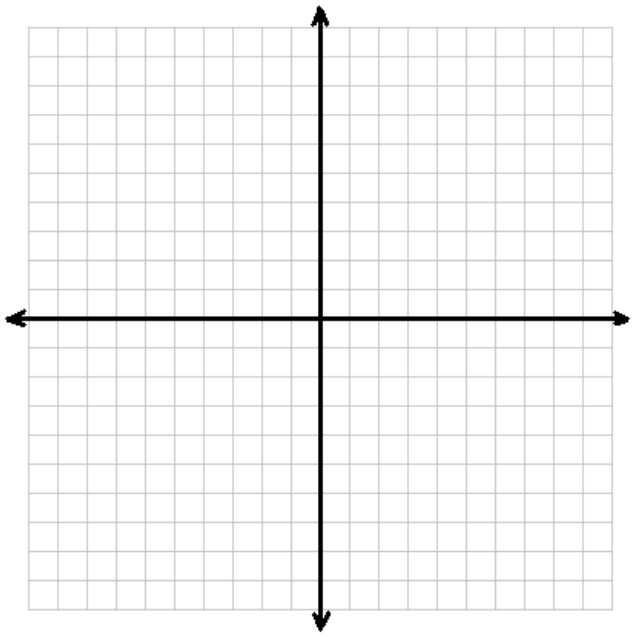
1. Rewrite the function replacing *f(x)* with *y*.
2. Interchange *x* and *y* in the equation defining the function.
3. Solve the new equation for *y*. If this equation cannot be solved uniquely for *y*, the original function has no inverse function.
4. Replace y with $f^{-1}(x)$

EXAMPLE: Find the inverse of:

a) $f(x) = 3x + 7$ b) $f(x) = 3x^3 - 2$

c) Verify that b above is the inverse by showing that $f(g(x))=x$ and $g(f(x))=x$.

EXAMPLE: Graph $y = \frac{2x-1}{3}$ and its inverse on the same axes.



Graphs of Inverse Functions

The graph of a function and its inverse are symmetric with respect to the line y = x.

Inverse Function on Limited Domain

If original function is not 1-to-1, the domain may reduced so that it becomes 1-to-1

Find the inverse of $f(x) = x^2 - 4$ for $x \geq 0$.

Example #46 The surface area of a cube is $y = 6x^2$ sq. cm, where x is the length of the edge of the cube in cm.

a. For what values of x does this model make sense? Is the model a 1-1 function for these values of x?

b. What is the inverse of this function on this interval?

c. How could the inverse function be used?

Homework Course Compass 4.3 and bookwork pages 290 #34, 48