Lesson 5.4

Objectives: To model with exponential functions
To find the constant percent change
To compare quadratic and exponential models
To model with logarithms

When the scatter plot of data shows a very rapid increase or decrease, it is possible that an exponential function can be used to model the data.

Exponential Growth

Example (#21) Total person income in the United States (in billions of dollars) for selected years from 1960 to 2005 is given in the following table.

a. These data can be modeled by an exponential function. Write the equation of this function, with x as the number of years after 1960.

b. If this model is accurate, what will be the total U.S. personal income in 2010?

c. In what year does the model predict the total personal income will reach $22 Trillion?

Year	1960	1970	1980	1990	2000	2005
Personal income	411.5	838.8	2307.9	4878.6	8429.7	10,239.2

Exponential Decay

In 1996, an industry wide program was instituted to reduce the number of on-the-job accidents. The table shows the success of the program for the period 1996-2000.

Year	1996	1997	1998	1999	2000
Accidents	50	29	18	11	7

a) If t is the number of years since 1996, find an exponential model for this data.

b) What would the model predict as the number of accidents in 2002? According to the model, can the number of accidents ever reach zero?

Constant Percent Changes

If the percent change of the outputs of a set of data is constant for equally spaced inputs, an exponential function will be a perfect fit for the data.

If the percent change of the outputs is approximately constant for equally spaced inputs, an exponential function will be an approximate fit for the data.

EXAMPLE: 1

Input	1	2	3	4	5	6
Output	4	16	64	256	1024	4096
1st differences		12	48	192	768	3072
% Change		12/4 = 300	48/16= 300	300	300	300

EXAMPLE: 2

Input	1	2	3	4	5	6
Output	1.5	2.25	3.8	5	11	17
1st differences		.75	1.55	1.2	6.	6
% Change		.75/1.5 = 50	1.55/2.25 = 69	32	55	

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EXAMPLE #3 The following table has input x and output g(x). Test the percent change of the outputs to determine if the function is exactly exponential, approximately exponential, or not exponential.

x	1	2	3	4	5	6
g(x)	2.5	6	8.5	10	8	6
1st differences						
% Change						

Exponential Model

If a set of data has initial value ***a*** and a constant percent change r (written as a decimal) for each equally spaced input x, the data can be modeled by the exponential function

$$y = a(l + r)^{x,}$$ for exponential growth

and $y = a(l - r)^{x}$ for exponential decay.

EXAMPLE: The average price of a house in a certain city is $220,000 in 2000, and it increases at 3% per year.

a. Write the equation of the exponential function that models the average price of a house *t* years after 2000.

b. Use the model to predict the average price of a house in 2010.

Comparison of Models

Sometimes it is difficult to determine what type of model is the best fit for a set of data. You can try graphing several different models and examining the model with the scatter plot and visually decide which is the best fit.

EXAMPLE The UN has projected the world population from 1995 to 2150 to be as shown in the table:

Year	1995	2000	2025	2050	2075	2100	2125	2150
Projected Population (millions)	5666	6113	9069	14,421	26,048	52,508	113,302	255,846

a. Find the exponential function that models the data, using $x = 0$ in 1990.

b. Find the quadratic function that is the best fit for the data, using $x = 0$ in 1990.

c. Graph each function on the same axes with the data points to determine which model is the better fit for the data.

Technology Note

When using technology to fit an **exponential model** to data, you should align the inputs to reasonably small values. When using technology to fit a **logarithm model** to data, you must align the data so that all input values are positive.

Logarithmic Models

Data that exhibits an initial rapid increase and then have a slow rate of growth can often be described by the function $f(x) = a + b \ln x$ for $b > 0$.

Data that exhibits an initial rapid decrease and then have a slow rate of decline can often be described by the function $f(x) = a + b \ln x$ for $b < 0$.

Note that the parameter a is the vertical shift of the graph of $y = \ln x$ and the parameter b affects how the grph of $y = \ln x$ is stretched.

Example#30 The percent of females in the workforce for selected years 1970-2006 are shown in the table below.

Year	1970	1975	1980	1985	1990	1995	2000	2005	2006
percent	30.0	31.9	35.3	37.9	39.2	40.3	41.3	41.3	41.5

a. Find a logarithmic function that models the data, using and input equal to the number of years after 1960.

b. When will the percent reach 43%, according to the model?

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