Lesson 6.5(F'09)

Objectives: To graph rational functions.
To find the vertical and horizontal asymptotes.
To solve rational equations analytically and graphically.

Rational Function

The function *f* is a rational function if $f(x)=\frac{P(x)}{Q(x)}$ where *P(x)* and *Q(x)* are polynomials and $Q(x)\neq 0$.

Vertical Asymptote

A vertical asymptote occurs in the graph of $f(x)=\frac{P(x)}{Q(x)}$ at those value of *x* where Q(x) = 0 and P(x) ≠ 0, or where Q(x) = 0 when the rational function is in lowest terms.

Find the equation of any (a) vertical asymptote:

#4 $f(x)=\frac{2x-5}{3-x}$

#6 $f(x)=\frac{x^2+6}{x^2+3}$

Horizontal Asymptote

If y approaches a as x approaches $+\infty$ or as x approaches $-\infty$, the graph of $y = f(x)$ has a horizontal asymptote at $y = a$.

Determining the Horizontal Asymptotes of a Rational Function

Consider the rational function

$$f(x)=\frac{P(x)}{Q(x)}=\frac{a_n x^n+\ldots+a_1 x+a_o}{b_m x^m+\ldots+b_1 x+b_o}$$

1. If $n < m$ (that is, the degree of the numerator is less than the degree of the denominator), a horizontal asymptote occurs as $y = 0$ (the x-axis)

2. If $n = m$ (that is, the degree of the numerator is equal to the degree of the denominator), a horizontal asymptote occurs at $y=\frac{a_n}{b_m}$

3. If $n > m$ (that is, the degree of the numerator is greater than the degree of the denominator), there is no horizontal asymptote.

Find the equation of any (a) vertical and (b) horizontal asymptote:

#4 $f(x)=\frac{2x-5}{3-x}$

#6 $f(x)=\frac{x^2+6}{x^2+3}$

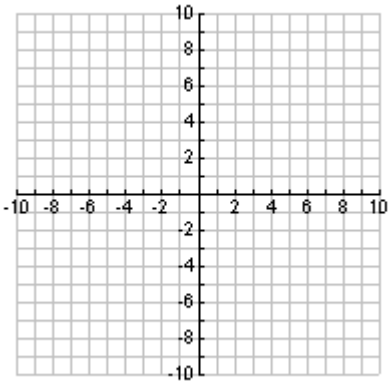
#8 Which of the following has a graph that does not have a horizontal asymptote? Why?

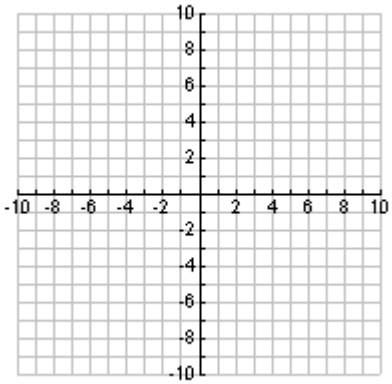
a) $f(x)=\frac{2x+3}{x-4}$ b) $f(x)=\frac{5x}{x^2-16}$

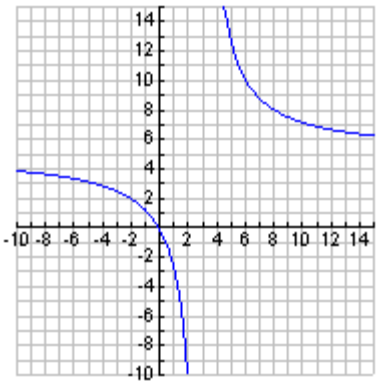
c) $f(x)=\frac{x^3}{3x^2+2}$ d) $f(x)=\frac{5}{(x+2)(x-3)}$

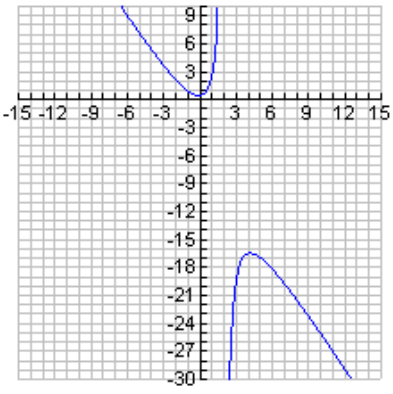
Find a) the horizontal asymptotes and b) the vertical asymptotes and c) sketch a graph of the function.

#10 $f(x)=\frac{5x}{x-3}$ #12 $f(x)=\frac{2x^2+1}{2-x}$



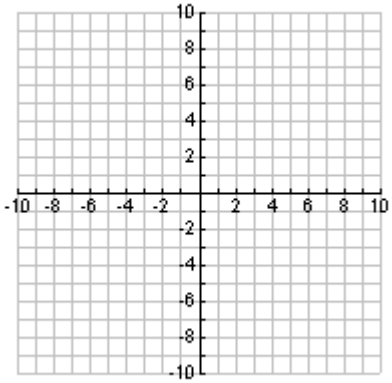






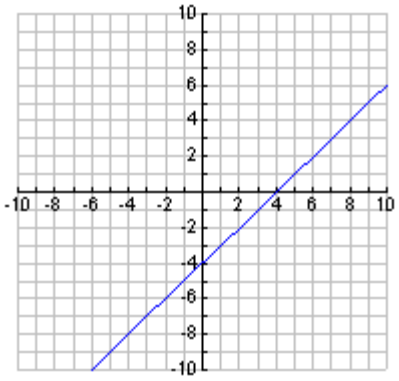
Some graphs have holes. Let's look at one example.

#14 Graph the function $f(x)=\frac{x^2-16}{x+4}$.



What happens at x = -4 ?

x	y1(x) (X^2-
-5	-9
-4	undef
-3	-7
-2	-6
-1	-5
0	-4
1	-3
2	-2
3	-1
4	0
5	1
6	2
7	3
8	4



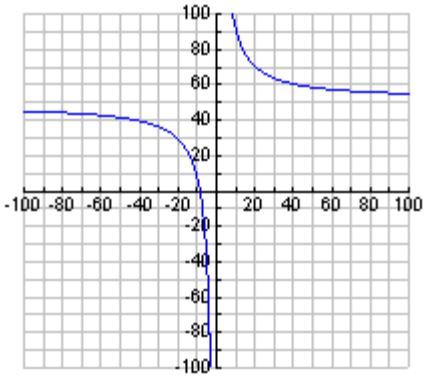
Applications

EXAMPLE (PAGE 495)#33 The average cost per unit for the production of a certain brand of DVD players is given by $\overline{C}=\frac{400+50x+0.01x^2}{x}$, where x is the number of hundred of units produced.

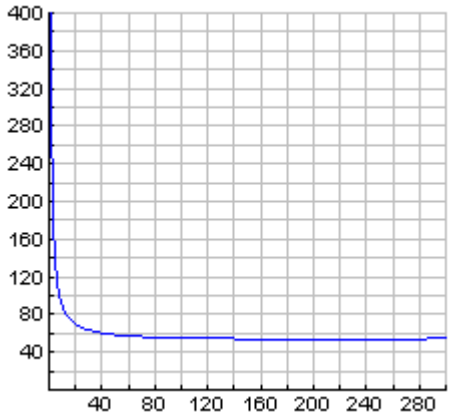
a. Graph this function on [-100,100] by [-100,100]

b. Graph this function on [0,300] by [0,400]

c. Which window is more appropriate for this problem? Why?



a

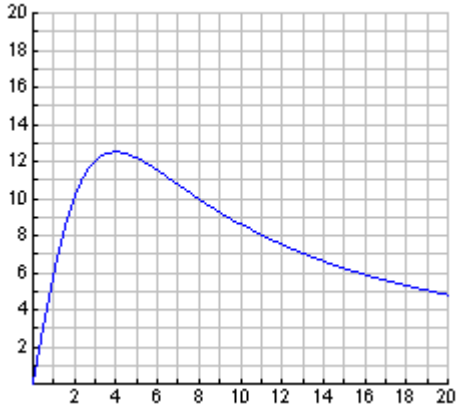


b

d. Use the graph to find the minimum average cost and the number of units that gives the minimum average cost.

EXAMPLE (page 495)#40 Suppose the concentration of a drug (as percent) in a patient's bloodstream t hours after injection is given by $C(t)=\frac{200t}{2t^2+32}$

a. Graph on [0,20] by [0,20]



b. What is the drug concentration 1 hour after injection?
5 hours?

c. What is the highest percent concentration?
In how many hours will it occur?

d. Describe how the end behavior of the graph of this function relates to the drug concentration.

EXAMPLE (page 4950)#42 The percent p of particulate pollution that can be removed from the smokestacks of an industrial plant by spending C dollars is given by $p = \frac{100C}{8300 + C}$

a. Find the percent of pollution that could be removed if spending were allowed to increase without bound.

b. Can 100% of the pollution be removed? Explain.

Analytical Solution of Rational Equation

1. Multiply both sides of the equation by the LCD of the fractions in the equation.

2. Solve the resulting polynomial equation for the variable analytically.

3. Check each solution in the original equations to eliminate any extraneous solutions.

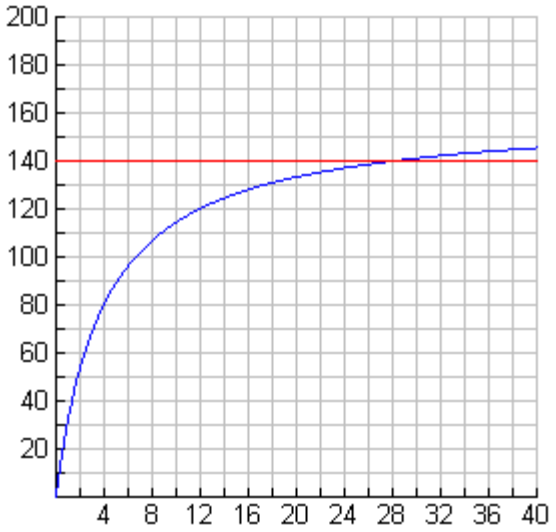
Solve Analytically

#22 d) $\frac{1}{4} = \frac{x^2}{(x+1)^2}$

#24 $\frac{x}{x-2} - x = 1 + \frac{2}{x-2}$

Solve Graphically

Weekly sales y (in hundreds of dollars) are related to weekly advertising expenses x (in hundreds of dollars) according to the equation $y = \frac{800x}{20+5x}$ Use graphical or numerical methods to find the amount of weekly advertising expenses that will result in weekly sales of $14,000, according to this model.



Homework Course Compass Section 6.5 page 493 #9-13, 39, 48 and the Supplement for Solving Rational Equations.

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