Lesson 6.1 & 6.2 (F'09)

Objectives: To graph cubic and quartic functions.
To find local and absolute minimum and maximum values with a calculator.
To find cubic and quartic models with a calculator.

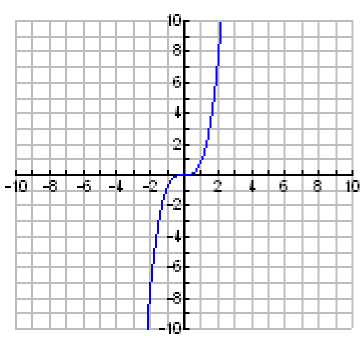
Cubic Functions

A cubic function in the variable x has an equation in the form:

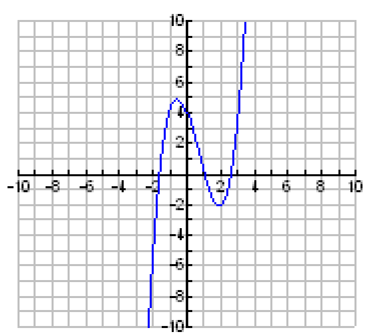
$$f(x) = ax^3 + bx^2 + cx + d \text{ with } a \neq 0.$$

In general, the graph of a polynomial function of degree n has at most **n** x-intercepts, and at most **n-1** turning points. Therefore a cubic equation has at most **3** x-intercepts, and at most **2** turning points. These points are called **local minimum and local maximum** points.

EXAMPLE:

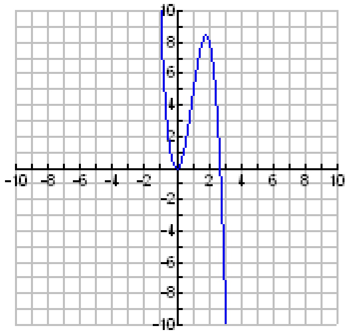


$y = x^3$



$y = x^3 - 2x^2 - 3x + 4$

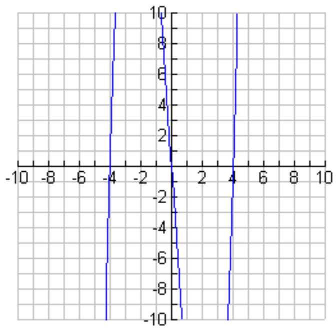
Note in the above graphs the ***a*** term is positive.



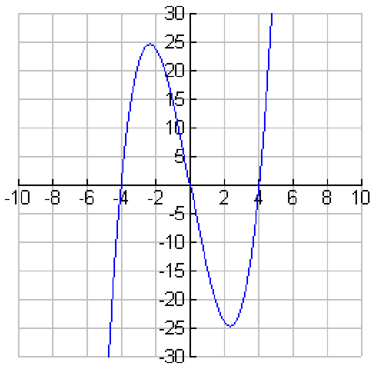
$y = -3x^3+8x^2$

Note in the above graph the ***a*** term is negative. Make sure to look at the table on page 382 in your book.

EXAMPLEGraph $y = x^3 - 16x$ using
a) The window [-10,10] by [-10,10]

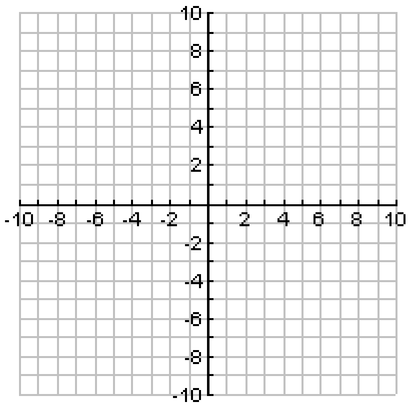


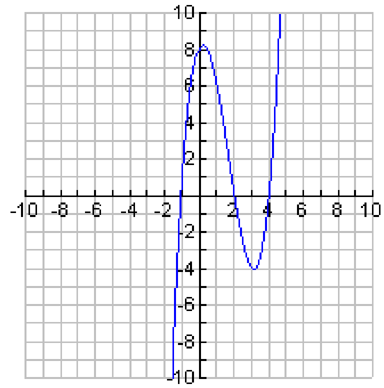
b) A window that shows two turning points.



c) Find the local mim. and max.

EXAMPLE(page 439): #30 Sketch a graph of any polynomial function that has degree 3, a positive leading coefficient, and three *x*-intercepts.

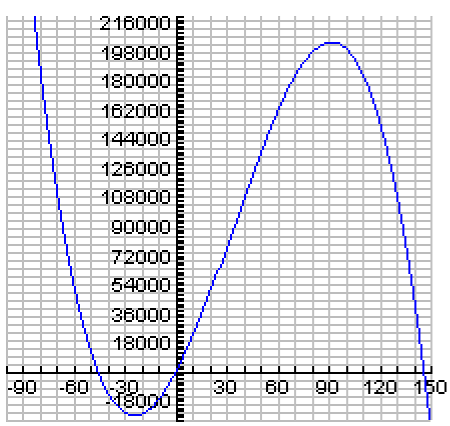




One possibility

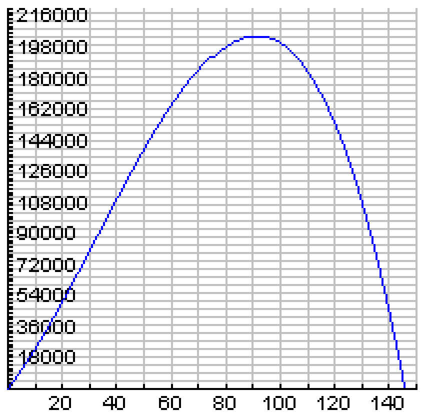
EXAMPLE(page 440)**:** #38 A firm has total weekly revenue for its product given by $R(x) = 2000x + 30x^2 - 0.3x^3$, where *x* is the number of units sold.

a. Graph on [-100,150] X [-30,000, 220,000]



b. Because *x* represents the number of units sold, what restrictions should be placed on *x* in the context of the problem? What restrictions should be place on *y*

c. Using your result from part (b), graph the function on a new window that makes sense for the problem.



d. What does this function give as the revenue if 60 units are produced?

EXAMPLE(page 442)#49 The annual revenue for a product is given by $R(x) = 60{,}000x - 50x^2$, and the annual cost is $C(x) = 800 + 100x^2 + x^3$, where x is the number of thousands for units produced and sold.

a. How many units will give maximum profit?

b. What is the maximum profit?

Your turn.

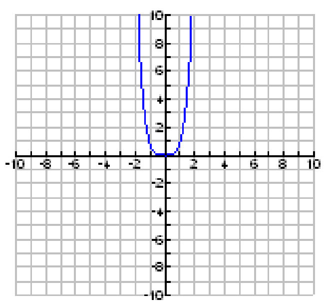
Example The annual revenue for a product is given by $R(s) = 60{,}000x - 50x^2$, and the annual cost is $C(s) = 800 + 100x^2 + x^3$, where x is the number is thousands of units produced and sold.

a. How may units will give the maximum profit?

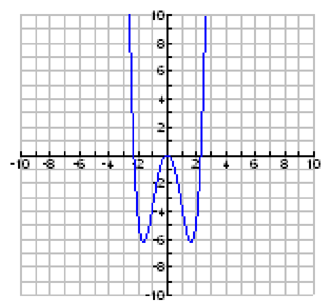
b. What is the maximum possible profit?

Quartic Function

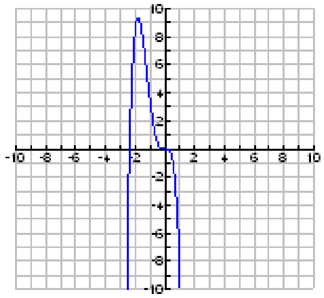
$f(x) = ax^4 + bx^3 + cx^2 + dx + e \quad a \neq 0$



$y = x^4$



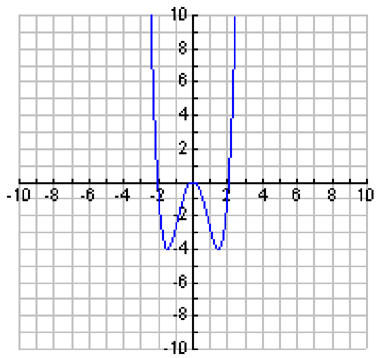
$y = x^4 - 5x^2$



$y = -3x^4 - 7x^3$

A quartic equation has: at most **4** x-intercepts.
at most **3** turning points.
an absolute maximum or minimum on the entire domain.

EXAMPLE Graph $y = x^4 - 4x^2$ making sure that you have a complete graph.



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Modeling (6.2)

Cubic Functions

Functions whose "third differences" are approximately the same may be modeled with a cubic.

EXAMPLE The number of births to females in the US under 15 years of age for certain years are shown in the accompanying table.

Year	Births (thousands)	Year	Births (thousands)
1960	6.780	1991	12.014
1970	11.752	1992	12.220
1980	10.169	1993	12.554
1986	10.176	1994	12.901
1990	11.657	1995	12.318

a. Find the cubic function that models the data, where y is the number of thousands of births and x is the number of years from 1960. Round the coefficients to four decimal places.

b. Use the rounded model reported in part (a) to estimate the number of such births that occurred in 1996.

Quartic

Functions whose "fourth differences" are approximately the same may be modeled with a quartic.

EXAMPLE(page398): #32 The annual changes in the Consumer Price Index (CPI) for 1990-2000 are shown in the following table.

Year	CPI Annual Change (%)	Year	CPI Annual Change (%)
1990	5.4	1996	3.0
1991	4.2	1997	2.3
1992	3.0	1998	1.6
1993	3.0	1999	2.2
1994	2.6	2000	3.4
1995	2.8		

a. Find the quartic function that models the data.

b. Is the fit of this model to the data best described as poor, good, or exact?
Look at the examples on pages 449-451 to see a comparison of the cubic and quartic models and the use of third and fourth differences.

Homework: Course Compass Toolbox chapter 6 and sections 6.1& 6.2 bookwork page 437 #2, 11-16, 29 and page 451 None

DUE BY________________________