Lesson 3.3 and 3.4 Fall 2009

Objectives: 1. To graph:
- a. Power functions
- b. Root functions
- c. Reciprocal Functions
- d. Piecewise functions

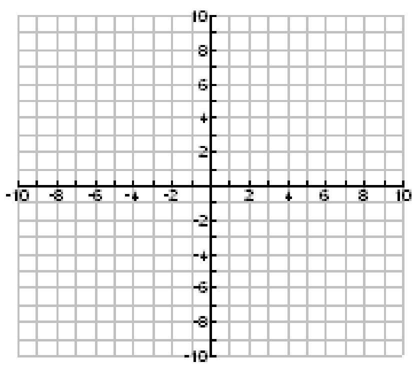
2. To model:
- a. Quadratic functions
- b. Power functions

3. To compare models.

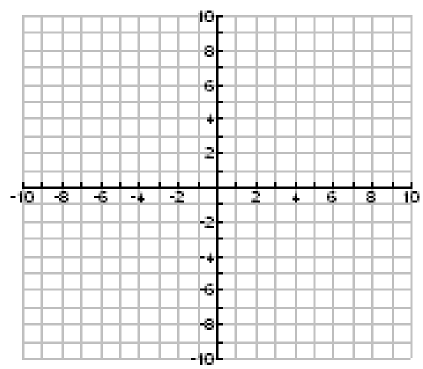
Power Function is a function of the form $y = ax^b$, where a and b are real numbers and $b \neq 0$.

Example: Graph each equation on [-10, 10] by [-10,10]

1. $y = x^3$



2. $y = x^{3/5}$



EXAMPLE #36The percent of all families that were single-parent families after 1960 was found to be modeled by $y = 1.053x^{0.888}$, with x = 0 in 1950.

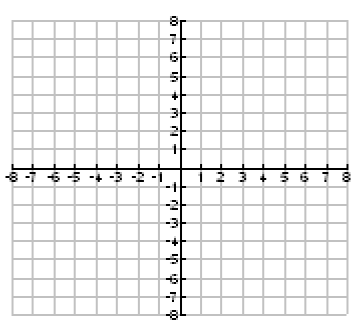
a. What is f(45)? What does this mean?

b. Does this model indicate that the percent of single families increased or decreased during this period after 1960?

Root Function a function of the form $y = ax^{1/n}$ or $y = a\sqrt[n]{x}$, where n is an integer, $n \geq 2$.

The graphs of these functions increase in the first quadrant but not as fast as y = x does for values of x greater that 1, so we say that they are concave down in the first quadrant.

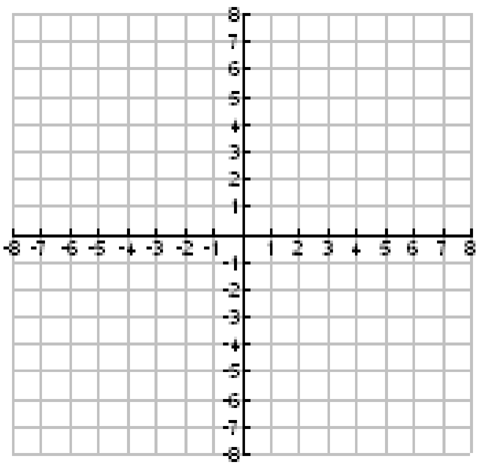
Graph $y = \sqrt[3]{x} + 2$ on [-8,8] by[-8,8]



<u>Reciprocal Function</u>

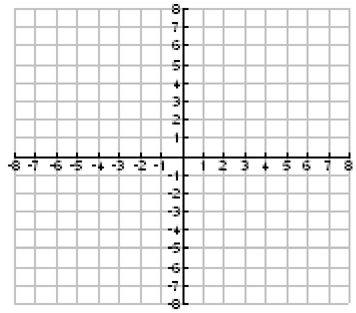
$y = \frac{1}{x}, x \neq 0$

Graph $y = \frac{1}{x} + 3$ on [-8,8] by[-8,8] (asymptote)



EXAMPLE #52 The monthly average cost of producing 27-inche television sets is $105 + \frac{50000}{x}$ dollars, where x is the number of sets produced per month. What is the average cost per set if 2000 sets are produced?

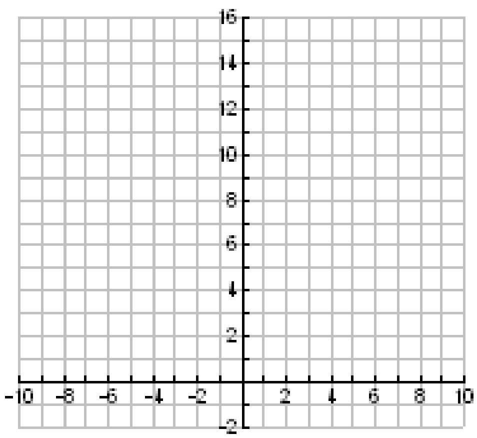
Piecewise-Defined Function



Graph 1. y = 5 if x ≥ 3

-2 if x < 3

2. $$y = \begin{cases} 3 - x & \text{if } x \leq 2 \\ x^2 & \text{if } x > 2 \end{cases}$$



EXAMPLE #42 The federal on-budget funds for all educational programs (in millions of constant 2000 dollars) between 1965 and 2000 can be modeled by the function $P(t) = \begin{cases} 1.965t - 5.65 & \text{when } 5 \le t \le 20 \\ 0.095t^2 - 2.925t + 54.429 & \text{when } 20 < t \le 40 \end{cases}$ where t is the number of years after 1960.

a. Graph the function for $5 \le t \le 40$. Describe how the funding for educational programs varied between 1965 and 2000.

b. What was the amount of funding for educational programs in 1980?

c. How much federal funding was allotted for educational programs in 1998?

Solving Absolute Value Equations

If $|x| = a$ and a > 0, then x = a or x = -a. There is no solution to $|x| = a$ = a if a < 0. V = 0 has solution x = 0.

Solve

a. $|x - 4| = 6$

b. $|x^2 - 5x| = 6$

Now let's graph quadratic and power models with our calculator.

Modeling with Quadratic Functions

Example: Use the following table to answer questions.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
	38	27	22	13	8	4	6	18	25	28	29

a. Create a quadratic function that models the data. We will do this just like a linear regression but we will use QuadReg.

   a. Enter the data into List 1 and list 2
   b. Then use Stat-cal #5
   c. Paste the function

b. Graph the aligned data and the quadratic function on the same axes. Does this model seem like a reasonable fit?

We can compare Linear and Quadratic Models by finding the differences of the y values when the x differences are constant.

Linear- First differences are fairly constant

Quadratic-Second differences are fairly constant

Example: #4 The following table has the inputs, x, and the outputs for three functions, f, g, and h. Use the second difference to determine which function is exactly quadratic, approximately quadratic or not quadratic.

x	f(x)	1st	2nd	g(x)	1st	2nd	h(x)	1st	2nd
0	0	399		2	-1.2		0	110	
2	399	1202	803	0.8	.4	1.6	110	51	-59
4	1601	1999	797	1.2	.2	1.6	161	34	-17
6	3600	2802	803	3.2	3.6	1.6	195	35	1
8	6402	3596	794	6.8	5.2	1.6	230	60	25
10	9998			12			290		

Example #18The percent of unemployment in the US for the years 2000-2007 is given by the data in the table below.

a. Create a scatter plot for the data, with x equal to the number of years after 2000.

b. Does it appear that a quadratic model will fit the data? If so, find the best-fitting quadratic model.

c. Does the y intercept of the function on part (b) have meaning in the context of this problem? If so, interrept the value.

Year	Percent unemployment	Year	Percent unemployment
2000	4.0	2004	5.5
2001	4.7	2005	5.1
2002	5.4	2006	4.8
2003	6.0	2007	4.6

Modeling with Powers ($y = ax^b$)

Example #35 The global spending on travel and tourism (in billions of dollars) for the years 1991-2005 is given in the table below.

a. Write the equation of a power function that models the data, letting your input represent the number of years after 1990.

b. Use the model to estimate the global spending for 2010.

c. When did the global spending reach $300 billion, according to this model?

Year	Spending	Year	Spending	Year	Spending
1991	278	1997	443	2003	533
1992	317	1998	445	2004	633
1993	323	1999	455	2005	682
1994	356	2000	483		
1995	413	2001	472		
1996	439	2002	487		

Comparison of Power and Quadratic Models

If data points appear to rise (or fall) more rapidly than a line, then a quadratic or power model may fit the data well.

Creating both models and comparing may be appropriate.

Sometimes the addition of another data point may clarify the best model.

Homework Course Compass Sections 3.3 and 3.4.
Bookwork page 213 #12, 14, 20 none from 3.4 and do the Piece Wise supplement