

## Lesson 3.1, 3.2 (Fall 2009)

- Objectives:
1. To graph quadratic functions (parabolas) by finding :
    - a) vertex (max or min)
    - b) intercepts
    - c) axis of symmetry
    - d) intervals of increase and decrease
  2. To solve quadratic equations.
  3. To solve application problems.

**Parabola-** is the graph of a quadratic function. A parabola has a turning point called the **vertex**.

### **General Form of a Quadratic Function-**

$$f(x) = ax^2 + bx + c \text{ for } a \neq 0. \quad \text{Vertex} \quad \left( \frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right).$$

The **domain** of a quadratic function is all real numbers.

The graph of a quadratic function opens up if  $a$  is positive, and is **concave up**. The vertex point is a **minimum**.

The graph of a quadratic function opens down if  $a$  is negative, and is **concave down**. The vertex is a **maximum**.

The vertical line through the vertex is called the **axis of symmetry** because it divides the graph into two halves that are reflections of each other.

## Increasing and Decreasing Functions

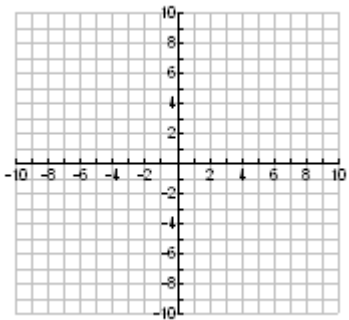
A function is increasing on an interval if, for in the interval, when  $x_2 > x_1$  it is true that  $f(x_2) > f(x_1)$ .

A function is decreasing on an interval if, for any  $x_1$  and  $x_2$  in the interval, when  $x_2 > x_1$  it is true that  $f(x_2) < f(x_1)$ .

Now let's look at two graphs.

EXAMPLE. Graph each parabola and fill in the blanks below.

1.  $y = x^2 + 1$



Vertex \_\_\_\_\_

Axis of symmetry \_\_\_\_\_

Opens \_\_\_\_\_

Increases \_\_\_\_\_

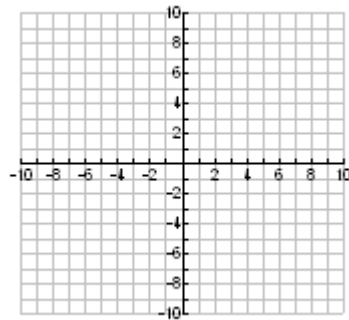
Decreases \_\_\_\_\_

Concave \_\_\_\_\_

Domain \_\_\_\_\_

Range \_\_\_\_\_

2.  $y = -x^2 + 3x - 4$



Vertex \_\_\_\_\_

Axis of symmetry \_\_\_\_\_

Opens \_\_\_\_\_

Increases \_\_\_\_\_

Decreases \_\_\_\_\_

Concave \_\_\_\_\_

Domain \_\_\_\_\_

Range \_\_\_\_\_

Role of  $a$  in the general equation.

$a = 1$       “normal”  
 $|a| > 1$      rises rapidly “thin”  
 $|a| < 1$      rises slowly “fat”

see summary page 176

### Standard Form of a Quadratic Equation

$y = a(x - h)^2 + k$  is a quadratic function with vertex at  $(h, k)$  and axis of symmetry  $x = h$ . ( Note the role of  $a$  is the same)

Example: Find the vertex and the equation of the axis of symmetry for each parabola. Is the vertex a max. or a min.?

1.  $f(x) = 2(x - 5)^2 + 3$

Vertex \_\_\_\_\_

Axis of symmetry \_\_\_\_\_

Max or Min \_\_\_\_\_

2.  $f(x) = -\frac{2}{3}(x + 6)^2 - 2$

Vertex \_\_\_\_\_

Axis of symmetry \_\_\_\_\_

Max or Min \_\_\_\_\_

Example: Find the equation of a parabola with the following points.

x	-1	0	1	2	3
y	13	-2	-7	-2	13

Remember to show a complete graph you must show:  
vertex, axis of symmetry, and the intercepts

Example: Graph  $y = -0.2x^2 - 4x + 18$  on your calculator.  
Find the x coordinate of the vertex, set your  
viewing window so that the vertex is near the  
center of the window.

Example: Suppose that the manufacturer of a gas clothes  
dryer has found that, when the unit price is p dollars, the  
revenue R (in dollars) is:  $R(p) = -4p^2 + 4000p$ .  
What unit price for the dryer should be established to  
maximize revenue? What is the maximum revenue?

Example: If 400 feet of fence are used to enclose a  
rectangular field, the resulting area of the field is  
 $A = x(200 - x)$ , where x is the width of the pen. What is the  
maximum area of the pen?

Example: The profit for a product is given by  $P = 1600 - 100x + x^2$ , where  $x$  is the number of units produced and sold. Graphically, find the  $x$ -intercepts of the graph of this function to find how many units will give break even (that is, return a profit of zero)

### Solving Quadratic Equations

Quadratic Equation in One Variable-  $ax^2 + bx + c = 0$  when  $a, b, c$  are real numbers and  $a \neq 0$ .

Factoring—Zero product Property

For real numbers  $a$  and  $b$ , the product  $ab = 0$  iff either  $a = 0$  or  $b = 0$ .

Example: Solve by factoring. Check by graphing.

1.  $3x^2 - 12x = 0$                       2.  $6x^2 - 13x + 6 = 0$

Example: Solve by graphing.

The solutions to  $ax^2 + bx + c = 0$  is(are) the x intercept(s) of  $y = ax^2 + bx + c$

1.  $0.1x^2 - 0.2x = 0.5$

Example: Solve using the

Square Root Property- If  $x^2 = a$  then  $x = \pm\sqrt{a}$  if  $\sqrt{a}$  is real.

1.  $x^2 = 45$

2.  $25x^2 = 49$

Example: Solve using the Quadratic formula

If  $ax^2 + bx + c = 0$ , then  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

1.  $2x^2 = x + 4$

2.  $6x^2 = -3x - 2$

Discriminant  $b^2 - 4ac$  if

a)  $b^2 - 4ac = 0$  \_\_\_\_\_

b)  $b^2 - 4ac > 0$  \_\_\_\_\_

c)  $b^2 - 4ac < 0$  \_\_\_\_\_

Example #52 The profit for a product is given by  $P(x) = -15x^2 + 180x - 405$  thousand dollars, where  $x$  is the number of tons of product produced and sold. How many tons give break-even (zero profit) for this product?

Example: #54 The total revenue function for a product is given by  $R = 266x$ , and the total cost function for this same product is  $C = 2000 + 46x + 2x^2$ , where  $R$  and  $C$  are each measured in thousands of dollars and  $x$  is the number of units produced and sold.

- a. Form the profit function for this product from the two given functions.
  
  
  
  
  
  
  
  
  
  
- b. What is the profit when 55 units are produced and sold?
  
  
  
  
  
  
  
  
  
  
- c. How many units must be sold to break even on this product?

Example. #60 The demand for a product is given by  $p = 7000 - 2x$  dollars, and the supply for this product is given by  $p = 0.01x^2 + 2x + 1000$  dollars, where  $x$  is the number of units demanded and supplied when the price per unit is  $p$  dollars. Find the equilibrium quantity and equilibrium price.

Homework Course Compass Toolbox 2 and sections 3.1 and 3.2 and bookwork page 182 # 12, 39, 49 and page 199 # 6, 35, 47, 55, and Supplement #1