

## Lesson 5.3 Part 2<sub>(Fall 2009)</sub>

Objectives: To find equations of exponential growth .  
To find equations of exponential decay.

In many situations that occur in science and economics a quantity changes at a rate proportional to the amount present. In these cases the amount at time  $t$  is a special function of  $t$ - called an exponential growth or decay function.

### Exponential Growth or Decay Function

Let  $y_0$  be the amount or number present at time  $t = 0$ . Then under certain conditions, the amount present at any time  $t$  is modeled by

$$y = y_0 e^{kt}, \text{ where } k \text{ is a constant.}$$

If  $k > 0$  this equation represents a growth function.

If  $k < 0$  this equation represents a decay function.

To find the equation of an exponential Growth or Decay function:

1. Start with the equation  $y = y_0 e^{kt}$ , and fill in the  $y_0$ ,  $y$ , and  $t$ .
2. Solve for  $k$ .
3. Now fill in the equation  $y = y_0 e^{kt}$  with  $y_0$ , and  $k$ .
4. This is your equation for this problem.
5. Now use this equation to answer the question.

Example: A colony of bacteria increases exponentially. If the number of bacteria doubles in 3 hours, find the function that gives the number of cells in the culture.

1. Since the number of bacteria doubles, let  $y_0 = 1$ ,  $y = 2$ . Because it takes 3 hours to double let  $t = 3$ . Our equation is:  $2 = (1) e^{3k}$
2. Solve for  $k$ .
3.  $k =$  \_\_\_\_\_
4. Our equation becomes  $y = (1)e^{\text{_____}t}$

How long will it take for the size of the colony to triple?

Replace  $y$  with 3 and solve for  $t$ .  $3 = (1) e^{0.231(t)}t =$  \_\_\_\_\_

Example: The population of a colony of mosquitoes increases exponentially. **a)** If there are 1000 mosquitoes initially and there are 1800 after 1 day, what is the size of the colony after 3 days?

$1800 = 1000e^{1k}$  Solve for  $k$ .  $k =$  \_\_\_\_\_

Now our equation is  $y = 1000e^{0.5877t}$  Replace  $t$  with 3 and find that there are \_\_\_\_\_ mosquitoes after 3 days.

**b)** How long is it until there are 10,000 mosquitoes?

Replace  $y$  with 10,000 and solve for  $t$ .  $10000 = 1000e^{0.5877t}$

$t =$  \_\_\_\_\_

Example: The half- life ( $y = 0.5$ ) of radium is 1690 years.

If 10 grams are present now, how much will be present in 50 years?

EXAMPLE: Boiling water, at  $100^{\circ}\text{C}$ , is placed in a freezer at  $0^{\circ}\text{C}$ . The temperature of the water is  $6^{\circ}\text{C}$  after 6 minutes. Find the temperature after 2 minutes.

Homework Handouts