M303 Homework Assignment 6

Please do the following problems but please do **not** hand them in:

(4.2) 1, 3, 5, 7, 9, 12abd, 13c, 16a
(5.1) 1, 3, 5, 7, 9, 11, 13

Answers for (4.2) 12: (a) $(\frac{3\sqrt{3}}{2}+2, \frac{3}{2}-2\sqrt{3})$ (b) $(\frac{7\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$ (c) $(4,3)$

Answer for (4.2) 16a: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$. Note, the matrix for a reflection about the line $y = x$ is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Please **hand in** the following problems Wednesday, October 7.

Section 4.2:

1. Consider the linear transformation $T(x_1, x_2) = (3x_1 - x_2,\ x_1 + 5x_2,\ 4x_1 - 6x_2)$.
   (a) What is the domain and codomain of this transformation?
   (b) Find the matrix for this transformation.
   (c) Let $\mathbf{x} = (7, 8)$. Find $T(\mathbf{x})$ by using the matrix you found in part (b). Check your answer by using the equation above to find $T(\mathbf{x})$ directly.

2. Use matrix multiplication to find the image of the vector $(3, 2)$ when it is rotated through an angle of
   (a) $\theta = 60^\circ$
   (b) $\theta = 270^\circ$

3. Let $T_1 : R^2 \to R^2$ be rotation by 90°. Let $T_2 : R^2 \to R^2$ be reflection about the x-axis.
   (a) Find the matrices for these two linear operators.
   (b) Find the matrix of the composite operator $T_2 \circ T_1$. That is, find the matrix for the following transformation: Rotate by 90° and then reflect about the x-axis.
   (c) Find the matrix of the composite operator $T_1 \circ T_2$. That is, find the matrix for the following transformation: Reflect about the x-axis and then rotate by 90°.

Section 5.1:

4. Let V be the set R^2 of real numbers, with the following operations: Let

$$(x, y) + (x', y') = \big(x + x', y + y'\big) \quad \text{and} \quad k(x, y) = (0, 0).$$

So V has the usual vector addition, but a peculiar scalar product. Determine if V is a vector space with these operations. If it is not a vector space, list all axioms that fail to hold.

5. Let V be the set R^2 of real numbers, with the following operations: Let

$$(x, y) + (x', y') = (2x + 2x', 2y + 2y') \quad \text{and} \quad k(x, y) = (kx, ky).$$

So V has the usual scalar product, but a peculiar vector addition. Determine if V is a vector space with these operations. If it is not a vector space, list all axioms that fail to hold.

6. Let V be the set of all 2×2 matrices of the form $\begin{bmatrix} a & a+b \\ 0 & b \end{bmatrix}$, given the usual matrix addition and scalar multiplication. Determine if this is a vector space. If it is not a vector space, list all axioms that fail to hold.