M303 Homework Assignment 1 Solutions

2. Solve the system by Gaussian elimination.

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & & & + & 3x_4 & = & -1 \\ x_1 & + & x_2 & - & 3x_3 & & & = & 3 \\ 2x_1 & - & 7x_2 & + & 3x_3 & + & 9x_4 & = & 0 \end{array}$$

In matrix form, this is:

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -1 \\ 1 & 1 & -3 & 0 & 3 \\ 2 & -7 & 3 & 9 & 0 \end{array}\right]$$

Subtract row one from row two:

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -1 \\ 0 & 3 & -3 & -3 & 4 \\ 2 & -7 & 3 & 9 & 0 \end{array}\right]$$

Subtract 2 times row one from row three:

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -1 \\ 0 & 3 & -3 & -3 & 4 \\ 0 & -3 & 3 & 3 & 2 \end{array}\right]$$

Now multiply row two by $\frac{1}{3}$:

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -1 \\ 0 & 1 & -1 & -1 & \frac{4}{3} \\ 0 & -3 & 3 & 3 & 2 \end{array}\right]$$

Now add 3 times row two to row three:

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -1 \\ 0 & 1 & -1 & -1 & \frac{4}{3} \\ 0 & 0 & 0 & 0 & 6 \end{array}\right]$$

But the last row of this matrix is the equation $0 = 6$, which is false. So the original system is inconsistent and has no solutions.

3. Solve the system by Gaussian elimination. (This is the same system as in the previous problem, but with one little change.)

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & & & + & 3x_4 & = & -1 \\ x_1 & + & x_2 & - & 3x_3 & & & = & 3 \\ -2x_1 & - & 7x_2 & + & 3x_3 & + & 9x_4 & = & 0 \end{array}$$

In matrix form, this is:

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -1 \\ 1 & 1 & -3 & 0 & 3 \\ -2 & -7 & 3 & 9 & 0 \end{array}\right]$$

First, subtract row one from row two, and add 2 times row one to row three:

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -1 \\ 0 & 3 & -3 & -3 & 4 \\ 0 & -11 & 3 & 15 & -2 \end{array}\right]$$

Now multiply row two by $\frac{1}{3}$:

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -1 \\ 0 & 1 & -1 & -1 & \frac{4}{3} \\ 0 & -11 & 3 & 15 & -2 \end{array}\right]$$

Now add 2 times row two to row one, and 11 times row two to row three:

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & \frac{5}{3} \\ 0 & 1 & -1 & -1 & \frac{4}{3} \\ 0 & 0 & -8 & 4 & \frac{38}{3} \end{array}\right]$$

Now multiply row three by $-\frac{1}{8}$:

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & \frac{5}{3} \\ 0 & 1 & -1 & -1 & \frac{4}{3} \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{19}{12} \end{array}\right]$$

Now add row three to row two, and add 2 times row three to row one:

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{3}{2} \\ 0 & 1 & 0 & -\frac{3}{2} & -\frac{1}{4} \\ 0 & 0 & 1 & -\frac{1}{2} & -\frac{19}{12} \end{array}\right]$$

This is equivalent to the system

$$\begin{aligned} x_1 \qquad\qquad\qquad\qquad &= -\tfrac{3}{2} \\ x_2 \qquad - \tfrac{3}{2}x_4 &= -\tfrac{1}{4} \\ x_3 \; - \tfrac{1}{2}x_4 &= -\tfrac{19}{12} \end{aligned}$$

So we let $x_4 = t$ and solve for the lead variables to obtain the solution:

$$\begin{aligned} x_1 &= -\tfrac{3}{2} \\ x_2 &= -\tfrac{1}{4} + \tfrac{3}{2}t \\ x_3 &= -\tfrac{19}{12} + \tfrac{1}{2}t \\ x_4 &= t \end{aligned}$$