

$A = \{a_1, a_2, a_3\}$  - number of rows =  $|A|$

$B = \{b_1, b_2, b_3, b_4, b_5\}$  - number of columns =  $|B|$

$R = ?$

	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$
$a_1$	0	1	0	0	0
$a_2$	1	0	1	1	0
$a_3$	1	0	1	0	1

1.  $R = ?$

2. Which are reflexive, symmetric, antisymmetric?

*Reflexive* relations have all 1's on the diagonal

*Symmetric* relation  $R$  iff  $\forall i \forall j m_{ij} = m_{ji}$

*Antisymmetric* relation  $R$  iff  $\forall i \forall j i \neq j \wedge m_{ij} \neq m_{ji}$

A	$b_1$	$b_2$	$b_3$
$a_1$	1	0	0
$a_2$	0	1	0
$a_3$	0	0	1

B	$b_1$	$b_2$	$b_3$
$a_1$	0	1	1
$a_2$	1	0	1
$a_3$	1	1	0

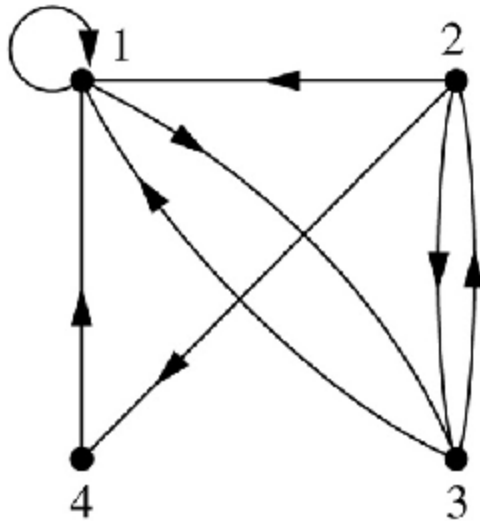
C	$b_1$	$b_2$	$b_3$
$a_1$	1	0	0
$a_2$	1	1	0
$a_3$	1	1	1

3.  $M_S \odot M_R$

$M_S$	1	2	3
1	1	0	1
2	0	0	0

$M_R$	1	2
1	1	0
2	0	1
3	0	0

4. E?



5. Is 1,3,1,1,2 a path?

Explain.

6. Give the graph of  
 $R = \{(1,2), (2,3), (3,4), (4,1), (3,1)\}$

7. Give  $R^{-1}$  and the symmetric closure of the relation.

$$A = \{1, 2, 3, 4\}$$

$$R = \{(1, 3), (1, 4), (2, 1), (3, 2)\}$$

	1	2	3	4
1	0	0	1	1
2	1	0	0	0
3	0	1	0	0
4	0	0	0	0

8. Find the zero-one matrix of the transitive closure of the relation R

$M_R$	1	2	3
1	1	0	0
2	0	1	0
3	1	1	0

$$M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]}$$

$A = \{a_1, a_2, a_3\}$  - number of rows =  $|A|$

$B = \{b_1, b_2, b_3, b_4, b_5\}$  - number of columns =  $|B|$

$R = ?$

	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$
$a_1$	0	1	0	0	0
$a_2$	1	0	1	1	0
$a_3$	1	0	1	0	1

- $R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3), (a_2, b_4), (a_3, b_1), (a_3, b_3), (a_3, b_5)\}$
- Which are reflexive, symmetric, antisymmetric?

*Reflexive* relations have all 1's on the diagonal

*Symmetric* relation  $R$  iff  $\forall i \forall j m_{ij} = m_{ji}$

*Antisymmetric* relation  $R$  iff  $\forall i \forall j i \neq j \wedge m_{ij} \neq m_{ji}$

Reflexive A,C  
Symmetric A,B  
Antisym. A, C

A	$b_1$	$b_2$	$b_3$
$a_1$	1	0	0
$a_2$	0	1	0
$a_3$	0	0	1

B	$b_1$	$b_2$	$b_3$
$a_1$	0	1	1
$a_2$	1	0	1
$a_3$	1	1	0

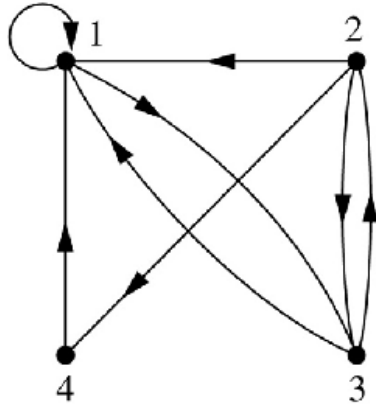
C	$b_1$	$b_2$	$b_3$
$a_1$	1	0	0
$a_2$	1	1	0
$a_3$	1	1	1

3.  $M_S \odot M_R$

$M_S$	1	2	3
1	1	0	1
2	0	0	0

$M_R$	1	2
1	1	0
2	0	1
3	0	0

	1	2
1	1	0
2	0	0

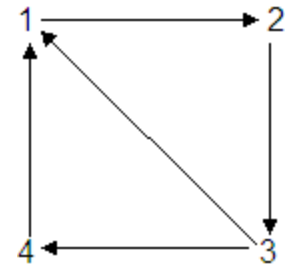


4.  $E?$

$$E = \{(1,1), (1,3), (2,1), (2,4), (2,3), (3,1), (3,2), (4,1)\}$$

5. Is 1,3,1,1,2 a path? No. Explain. No 1,2 path

6. Give the graph of  $R = \{(1,2), (2,3), (3,4), (4,1), (3,1)\}$



7. Give  $R^{-1}$  and the symmetric closure of the relation.  $A = \{1,2,3,4\}$

$$R = \{(1,3), (1,4), (2,1), (3,2)\}$$

$$R^{-1} = \{(3,1), (4,1), (1,2), (2,3)\}$$

	1	2	3	4
1	0	1	1	1
2	1	0	1	0
3	1	1	0	0
4	1	0	0	0

8. Find the zero-one matrix of the transitive closure for R

$$M_R^* = \begin{matrix} & M_R^* & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{matrix} = \begin{matrix} & M_R & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{matrix} \vee \begin{matrix} & M_R^{[2]} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{matrix} \vee \begin{matrix} & M_R^{[3]} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{matrix}$$