0. Give the search space for each iteration on:

binary search( 50, { 10, 16, 27, 38, 49, 53, 76, 82, 85, 91, 98 } )

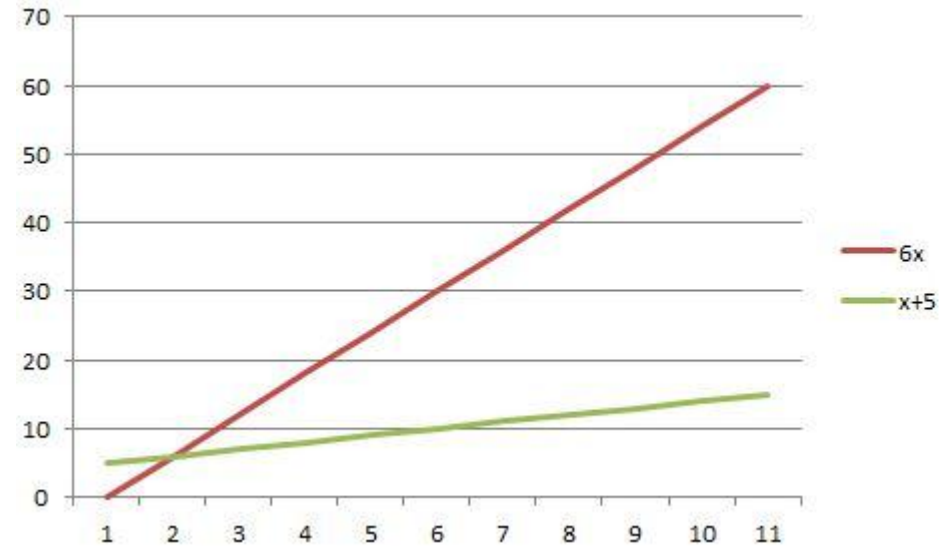
**procedure** binary search(x: **integer**, $a_1 < a_2 < ... < a_n$: integers)
l := 1
r := n
**while** l < r

m := $\lfloor (l + r) / 2 \rfloor$

**if** x > $a_m$ **then**
l := m + 1
**else**
r := m
**if** x = $a_l$ **then**
loc := l
**else**
loc := 0

1. Graph $f(x) = x+6$
   $f(x) = 3x$

| x | x+6 | 3x |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |




Which dominates the other? At what value of x?

Which of these functions is O(x)?

2. $f(x) = 20x+56$
3. $f(x) = x/4$
4. $f(x) = x^2$
5. $f(x) = x \log x$

6. Derive C and k for $f(x)=(x^2+1)$ to show $O(x^2)$

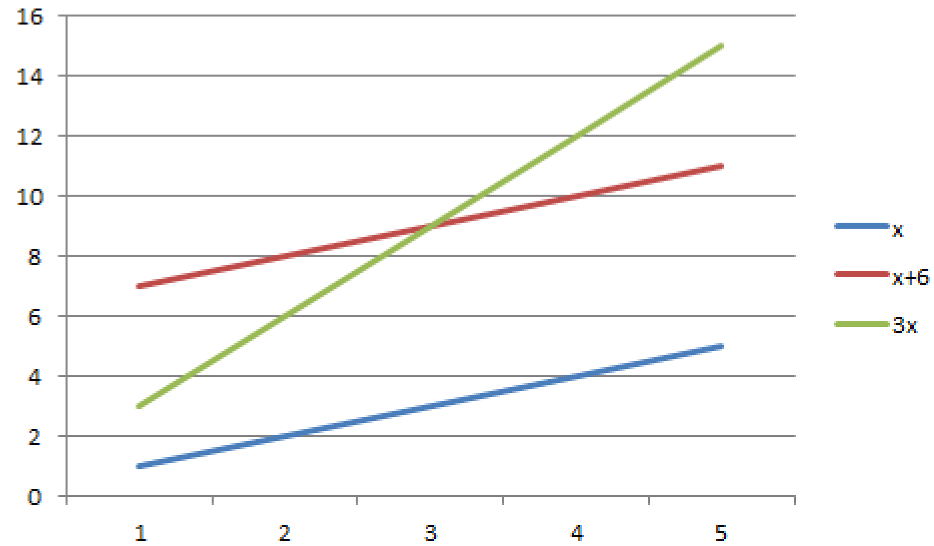
0. Give the search space for each iteration on:

binary search( 50, { 10, 16, 27, 38, 49, 53, 76, 82, 85, 91, 98 } )

| | |
|---|---|
| **procedure** binary search(x: **integer**, $a_1 < a_2 < ... < a_n$: integers) | |
| l := 1 | |
| r := n | l=1, r=11, m=6 |
| **while** l < r | { 10, 16, 27, 38, 49, 53, 76, 82, 85, 91, 98 } |
| | l=1, r=6, m=3 |
| m := $\lfloor (l + r) / 2 \rfloor$ | { 10, 16, 27, 38, 49, 53, 76, 82, 85, 91, 98 } |
| | l=4, r=6, m=5 |
| **if** x > $a_m$ **then** | { 10, 16, 27, 38, 49, 53, 76, 82, 85, 91, 98 } |
| l := m + 1 | l=5,r=6,m=5 |
| **else** | { 10, 16, 27, 38, 49, 53, 76, 82, 85, 91, 98 } |
| r := m | l=6,r=6,m=6 |
| **if** x = $a_l$ **then** | { 10, 16, 27, 38, 49, 53, 76, 82, 85, 91, 98 } |
| loc := l | |
| **else** | |
| loc := 0 | |

1. Graph $f(x) = x+6$ and $f(x) = 3x$

| x | x+6 | 3x |
|---|---|---|
| 1 | 7 | 3 |
| 2 | 8 | 6 |
| 3 | 9 | 9 |
| 4 | 10 | 12 |
| 5 | 11 | 15 |




Which dominates? 3x At what point? x=3

Which of these functions is O(x).

2. $f(x) = 20x+56$ Yes
3. $f(x) = x/4$ Yes
4. $f(x) = x^2$ No
5. $f(x) = x \log x$ No

5. Derive C and k for $f(n)=n^2+1$ to show $O(n^2)$

| | |
|---|---|
| $f(n)=n^2+1$ | $g(n)=n^2$ |
| | |
| $f(n) \le C * g(n)$ | From definition of Big-O |
| $n^2+1 \le C\, n^2$ | substitute for f(n) and g(n) |
| $(n^2+1)/n^2 \le C$ | divide both sides by $n^2$ |
| $n^2/n^2+1/n^2 \le C$ | |
| $1+1/n^2 \le C$ | maximum at n=1 |
| $C = 2$ | |
| | |
| $1+1/k^2 \le 2$ | Substitute C=2 and k |
| $1/k^2 \le 1$ | Subtract |
| $1 \le k^2$ | |
| $1 \le k$ | Square root |
| $k = 1$ | |
| | |
| $n^2+1 \le 2n^2,\ n > 1$ | Satisfies $f(n) \le C * g(n)$ |