Translate each to English, where the domain consists of all real numbers.

1. $\forall x \exists y (x < y)$
2. $\forall x \forall y ((x \geq 0 \wedge y \geq 0) \rightarrow xy \geq 0)$

Express so that negation immediately precedes predicate.

3. $\neg \forall x \exists y P(x,y)$
4. $\neg \forall x \forall y (P(x) \wedge F(y))$

What rule of inference is used in the following?

4. Alice is a math major. Therefore, Alice is a math or CS major.

5. If it snows today then school will close. School is not closed. Therefore it did not snow today.

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \land (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \land (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \land (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$[(p \lor q) \land \neg p] \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$[(p) \land (q)] \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$[(p \lor q) \land (\neg p \lor r)] \to (q \lor r)$	Resolution

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\forall x P(x)$ ∴ $P(c)$	Universal instantiation
$P(c)$ for an arbitrary c ∴ $\forall x P(x)$	Universal generalization
$\exists x P(x)$ ∴ $P(c)$ for some element c	Existential instantiation
$P(c)$ for some element c ∴ $\exists x P(x)$	Existential generalization

For x=-1,0,1.

6. Given ∀xP(x)
   What is P(-1)?

7. Given P(c) for any c
   What is ∀xP(x)?

Definition of even integer: $n = 2k$
Definition of odd integer: $n = 2k+1$

The sum of two even integers is even.

8. Restate as $p \rightarrow q$.

9. What is the hypothesis, p?

10. What is the conclusion, q?

11. Give a direct proof.

Definition of even integer: $n = 2k$
Definition of odd integer: $n = 2k+1$

If n is an even number then n^2 is an even number.

12. What is the hypothesis, p?

13. What is the conclusion, q?

14. What is the contrapositive?

15. Give a direct proof.

If n^2 is an odd integer then n is an odd integer.

16. Prove by contradiction.

Translate each to English, where the domain consists of all real numbers.

1. $\forall x \exists y (x < y)$ **For all x there exits a y such that x<y**
2. $\forall x \forall y ((x \geq 0 \wedge y \geq 0) \rightarrow xy \geq 0)$ **For all x and for all y, if x≥0 ^ y≥0) then xy≥0**

Express so that negation immediately precedes predicate.

3. $\neg \forall x \exists y P(x,y) = \exists x \neg \exists y P(x,y) = \exists x \forall y \neg P(x,y)$
4. $\neg \forall x \forall y (P(x) \wedge F(y)) = \exists x \neg \forall y (P(x) \wedge F(y)) = \exists x \exists y \neg (P(x) \wedge F(y)) = \exists x \exists y (\neg P(x) \vee \neg F(y))$

What rule of inference is used in the following?

4. Alice is a math major. Therefore, Alice is a math or CS major.
Addition

5. If it snows today then school will close. School is not closed. Therefore it did not snow today.
Modus tollens

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$[p \wedge (p \to q)] \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$[\neg q \wedge (p \to q)] \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \to q$	Disjunctive syllogism
p $\therefore p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Simplification
p q $\therefore p \wedge q$	$[(p) \wedge (q)] \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$[(p \vee q) \wedge (\neg p \vee r)] \to (q \vee r)$	Resolution

For x=-1,0,1.

6. Given $\forall x P(x)$
   What is P(-1)
   True

7. Given P(c) for any c
   What is $\forall x P(x)$
   True

TABLE 2 Rules of Inference for Quantified Statements.

Rule of Inference	*Name*
$\forall x P(x)$ $\therefore P(c)$	Universal instantiation
$P(c)$ for an arbitrary c $\therefore \forall x P(x)$	Universal generalization
$\exists x P(x)$ $\therefore P(c)$ for some element c	Existential instantiation
$P(c)$ for some element c $\therefore \exists x P(x)$	Existential generalization

The sum of two even integers is even.

8. Restate as $p \rightarrow q$.
   If x and y are even integers, then x+y is even.
9. What is the hypothesis, p?
10. What is the conclusion, q?
11. Give a direct proof.

Prove by direct proof
"If x and y are even integers, then x+y is even"
$\forall n\ (P(n) \rightarrow Q(n))$
Hypothesis
“x and y are even integers"
Show (conclusion) is true
“x+y is even."

P(n)	Assume hypothesis, x and y even: x = 2i, y=2j
x+y = 2i+2j	Definition of even x and y
= 2(i+j)	Definition of even

$\therefore$ even x+y

Definition of even integer: $n = 2k$
Definition of odd integer: $n = 2k+1$
If n is an even number then n^2 is an even number.

12. What is the hypothesis, p?
13. What is the conclusion, q?
14. What is the contrapositive?

If n^2 is an odd number then n is an odd number .

15. Give a direct proof.

Prove by direct proof
"If n is an even integer, then n^2 is even"

$\forall n\,(P(n) \rightarrow Q(n))$

Hypothesis

"n is an even integer"

Show (conclusion) is true

"Given any even integer n, n^2 is even."

P(n)	Assume hypothesis, n is even: $n = 2k$
$n = 2k$	Definition of even n
$n^2 = (2k)^2$	Square both sides
$(2k)^2 = 4k^2$	Multiply
$4k^2 = 2(2k^2)$	Factor
$n^2 = 2(2k^2)$	Definition of even integer, 2k.

$\therefore$ n^2 is even

Definition of even integer: $n = 2k$
Definition of odd integer: $n = 2k+1$

If n^2 is an odd integer then n is an odd integer.

16. Prove by contradiction.

Prove by contradiction
"If n is an odd integer, then n^2 is odd"

$$\forall n\ (P(n) \rightarrow Q(n))$$

Hypothesis, p:"n^2 is an odd integer"
Conclusion, q: "n is odd integer.“
⌐q: n is even integer.

$n = 2k$	Assume ⌐q, definition of even n
$n^2 = (2k)^2$	Square both sides
$(2k)^2 = 4k^2$	Multiply
$4k^2 = 2(2k^2)$	Factor
$n^2 = 2(2k^2)$	Definition of even integer, 2k, contradicts hypothesis, n^2 is odd.

$\therefore$ n is odd