9.4 Geometry in Three Dimensions

SIMPLE CLOSED SURFACES have exactly one interior, no holes and are hollow.
Some examples: the exterior of a silo
a soup can
a basketball, which is a sphere (defined to be the set of all points at a given distance from a given point, the center)

A simple closed surface partitions space into three distinct sets:
Points outside the surface, points inside the surface, and points on the surface.

The union of all points on a simple closed surface and all interior points is called a solid.

Look at figure 9-35 on page 615. Which of these figures are simple closed surfaces?________________
Which are not? _________________ Why not?____

We are going to study several types of simple closed surfaces in this section.

POLYHEDRON: A simple closed surface made up of polygonal regions.
$Poly$→many $Hedron$→flat surface

Look again at figure 9-35 and decide which of these are polyhedra ($plural\ of\ polyhedron$).

Each of the polygonal regions of a polyhedron is called a face. The vertices of the polygonal regions are the vertices of the polyhedron. The sides of the polygonal regions are the edges of the polyhedron.

PRISM: A polyhedron in which 2 congruent polygonal faces lie in parallel planes and the other faces are bounded by parallelograms. The 2 congruent polygonal faces are called the bases of the prism, and the prism is usually named after its bases. The other faces are called the lateral faces.

Look at figure 9-36 on page 615.
In figure a), the bases are__________________________
Can you tell why they labelled this a Triangular Right Prism?
What is the shape of its lateral faces?________________________

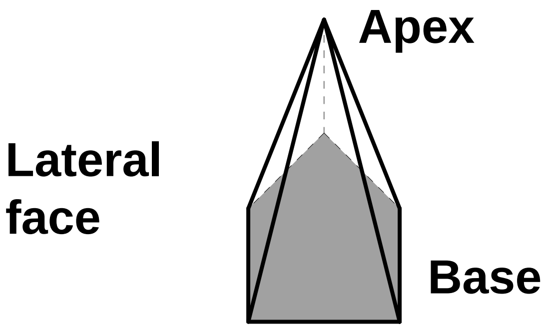
Figures b) and c) are both right prisms also. What is the shape of their lateral faces?__________________

What is the difference between c) and d)?

Note: This author gives you tips for 3-D drawing along the way.

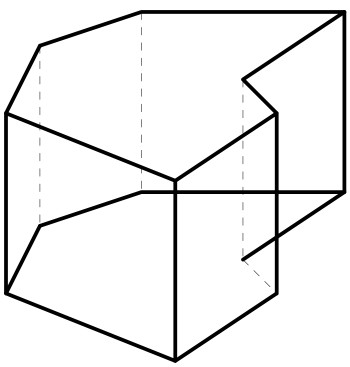
PYRAMID: A polyhedron determined by:
1) a simple closed polygonal region (called the base);
2) a point not in the plane of this region (calles the apex); and
3) triangular regions determined by the point and each pair of consecutive vertices of the polygonal region (called the lateral faces).

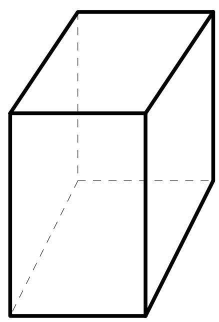
Here is a pentagonal pyramid:



See figure 9-38 on page 616 for 2 other pyramids.

CONVEX POLYHEDRA: A polyhedron is convex iff the segment joining any 2 points in the interior of the polyhedron is itself in the interior. Otherwise the polyhedron is concave.
Classify these as either convex or concave:





REGULAR POLYHEDRA: A convex polyhedron whose faces are congruent, regular, polygonal regions such that the number of edges that meet at each vertex is the same for all the vertices of the polyhedron.

Can you think of an everyday object that is a regular polyhedron?
Draw its sketch and count the number of edges that meet at each vertex_______________

How many regular polyhedra are there? This problem is addressed on page 618.
Let's look at the solution.

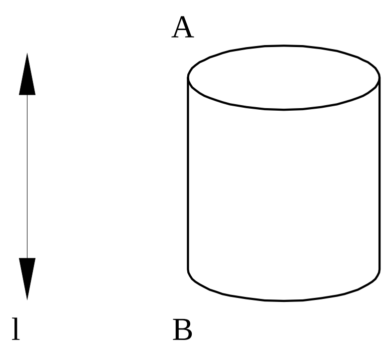
Polygonal Face:	**Measure of Interior angle:**	**No. of Polygons at a Vertex:**	**Sum of Angles at the vertex:**	**Possible? < 360°:**	**Polyhedron Formed:**
Triangle	60°	3	180°	Yes	Tetrahedron
Triangle	60°	4	240°	Yes	Octahedron
Triangle	60°	5	300°	Yes	Icosahedron
Triangle	60°	6	360°	No	
Square	90°	3	270°	Yes	Cube
Square	90°	4	360°	No	
Pentagon	108°	3	324°	Yes	Dodecahedron
Pentagon	108°	4	432°	No	
Hexagon	120°	3	360°	No	

So, there are five regular polyhedra.

Euler's Formula describes the relationship among the number of vertices (V), faces (F) and edges (E) of any polyhedron. Can you use the information in the following table to come up with this formula relating V, F and E?

Polyhedron	**V**	**F**	**E**
Tetrahedron	4	4	6
Cube	8	6	12
Octahedron	6	8	12
Dodecahedron	20	12	30
Icosahedron	12	20	30

CYLINDERS A cylinder is a simple closed surface, but it is not a polyhedron.
Let's look at how a cylinder is formed.
Start with a line l and a line segment $\overline{AB}$. If the line segment moves so that iti is always parallel to the line, and if its endpoints A and B trace simple closed planar curves other than polygons, the surface generated by $\overline{AB}$ and the simple closed curves and their interiors form a cylinder. The simple closed curves are called the bases and the remaining points make up the lateral surface of the cylinder.



This is a right circular cylinder because the base is a circle and $\overline{AB} \perp$ the base of the cylinder.

Cylinders that are not right cylinders are called oblique cylinders. See figure 9-43 on page 622 b) and c)

CONES To form a cone, you need a simple closed planar curve and a point that is not in the same plane as this curve. Then consider connecting line segments from the point to that curve. The point is called the vertex and the simple closed planar curve, the base of the cone. The points of the cone that are not in the base constitute the lateral surface of the cone. A line segment from the apex perpendicular to the plane of the base is called the altitude. A right circular cone is a cone whose altitude intersects its circular base at its center. See figure 9-44 page 622 which shows cones that are oblique, as well as a right circular cone.